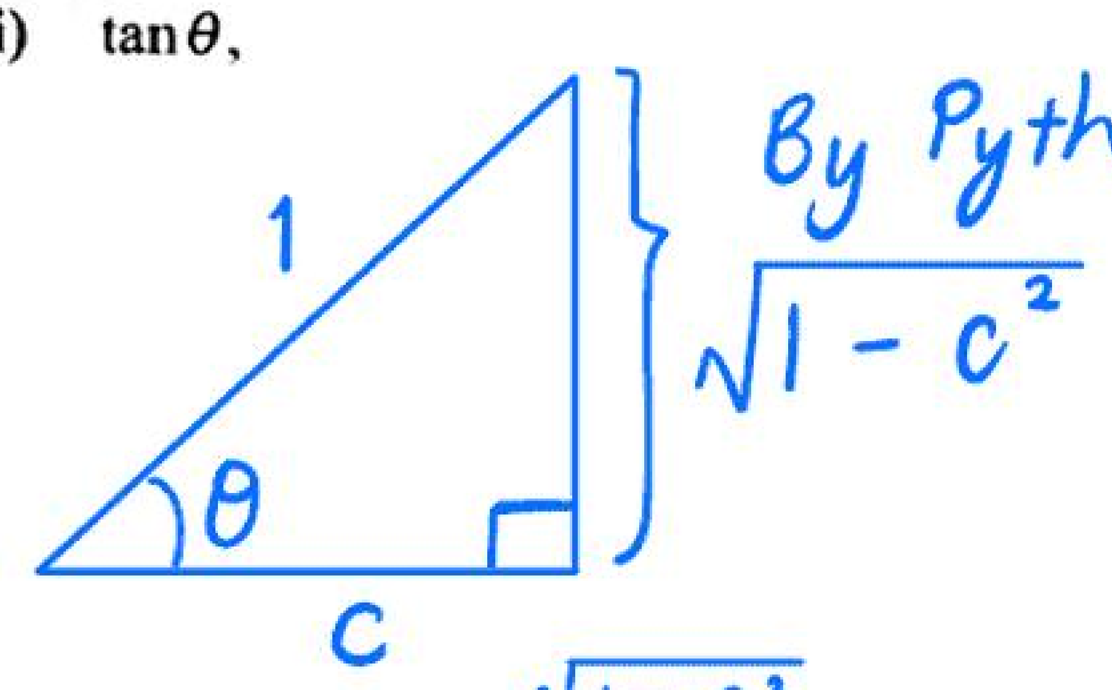
1 Given that θ is acute and $\cos\theta = c$, express, in terms of c,

(i) $\tan\theta$, [3]



By Pythagoras' Theorem,

$\therefore \tan\theta = \dfrac{\sqrt{1-c^2}}{c}$

(ii) $\operatorname{cosec}\theta$. [1]

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\sqrt{1-c^2}}$

2 Find the set of values of the constant k for which the curve $y = x^2 + (2k+1)x + 1$ lies entirely above the line $y = x$. [4]

$y = x^2 + (2k+1)x + 1$ —— (1)

$y = x$ —— (2)

Sub (2) into (1):

$x^2 + (2k+1)x + 1 = x$

$x^2 + 2kx + 1 = 0$

$b^2 - 4ac < 0$

$(2k)^2 - 4 < 0$

$4k^2 - 4 < 0$

$k^2 - 1 < 0$

$(k-1)(k+1) < 0$

$\therefore -1 < k < 1$

since $y = x^2 + (2k+1)x + 1$ lies entirely above the line $y = x$, there is NO Intersection i.e. No real roots

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3 Given that $y = Ae^{2x} + Be^{-x}$, and that $\frac{dy}{dx} + 4y = e^{2x} - e^{-x}$, find the value of each of the constants A and B. [4]

Given $y = Ae^{2x} + Be^{-x}$ ——(1)

$\therefore \frac{dy}{dx} = 2Ae^{2x} - Be^{-x}$ ——(2)

Also, given that $\frac{dy}{dx} + 4y = e^{2x} - e^{-x}$ ——(3)

Sub (1) and (2) into (3):

$2Ae^{2x} - Be^{-x} + 4Ae^{2x} + 4Be^{-x} = e^{2x} - e^{-x}$

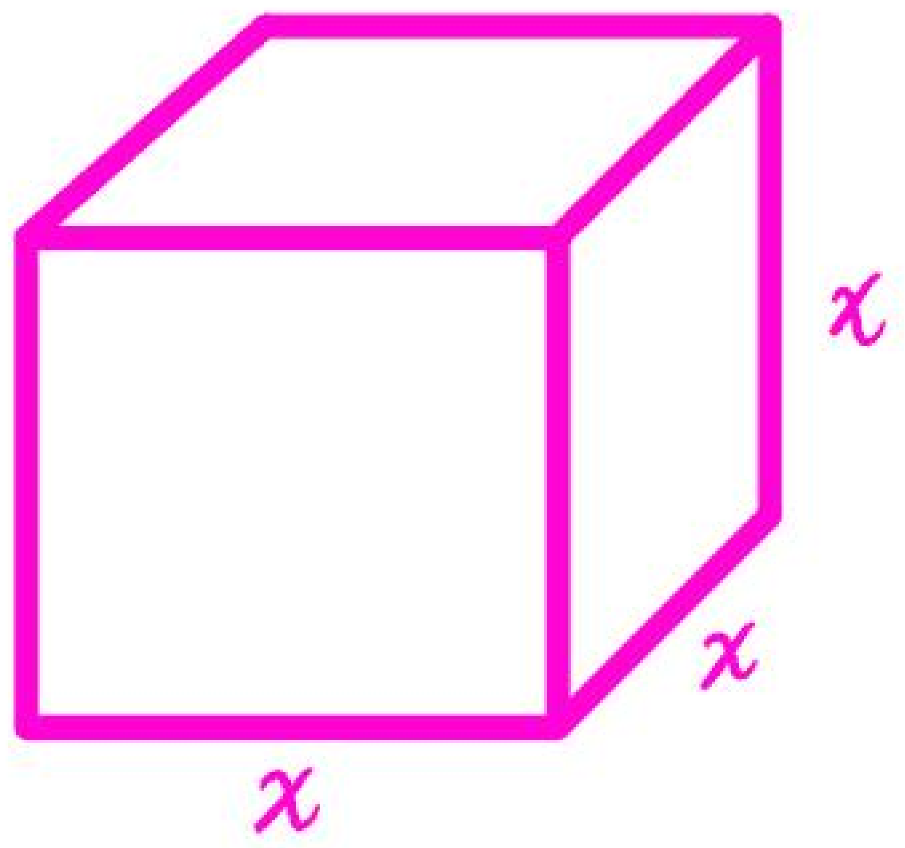
$6Ae^{2x} + 3Be^{-x} = e^{2x} - e^{-x}$

By comparison of coefficients;

$6A = 1$, $3B = -1$

$A = \frac{1}{6}$ $B = \frac{-1}{3}$

4 An ice cube of side x cm is melting in such a way that the total surface area, $A\,\text{cm}^2$, is decreasing at a constant rate of $48\,\text{cm}^2/\text{s}$. Assuming that the cube retains its shape, calculate the rate of change of x when $x = 10$. [4]



$A = 6x^2$

$\frac{dA}{dx} = 12x$

Given: $\frac{dA}{dt} = -48\ \text{cm}^2/\text{s}$

Find: $\frac{dx}{dt}$ when $x = 10$ cm

Missing: $\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}$

$-48 = 12(10) \cdot \frac{dx}{dt}$

$\therefore \frac{dx}{dt} = \frac{-48}{120} = \underline{-0.4\ \text{cm/s}}$

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5 **(i)** A manufacturer produces a disinfectant that destroys 21% of all known germs within one minute of use. If N is the number of germs present when the disinfectant is first used, and assuming germs continue to be destroyed at the same rate, explain why the number of germs expected to be alive after n minutes is given by $(0.79)^n N$. [2]

when $t = 0$ mins, number of germs $= N$
after 1 min, number of germs $= 0.79N$
after 2 mins, number of germs $= 0.79(0.79N)$
$= 0.79^2 N$

⋮

after n mins, number of germs present $= \underline{0.79^n N}$

(ii) The manufacturer decides to advertise by stating that the disinfectant destroys x% of all known germs within 20 minutes of use. Calculate, to 2 significant figures, the value of x. [2]

after 20 mins, number of germs present $= 0.79^{20} N$

$$\frac{N - 0.79^{20}N}{N} \times 100\% = x\%$$

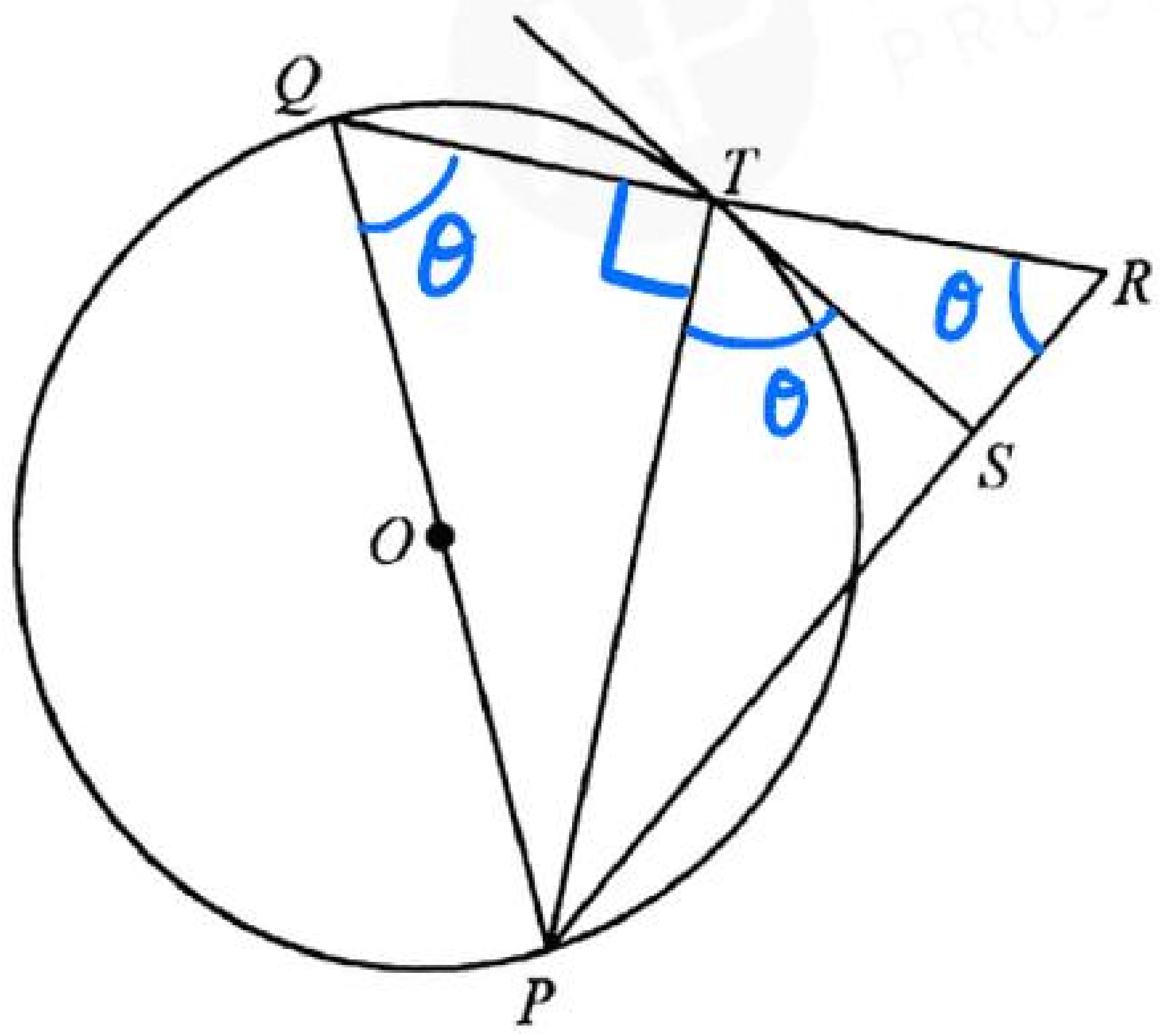
$$\frac{\cancel{N}(1-0.79^{20})}{\cancel{N}} \times 100 = x$$

$\therefore x = 99.104$
$= \underline{99}$ (correct to 2 s.f.)

(iii) Given that the number of germs expected to be alive after n minutes can be expressed as Ne^{kn}, find the value of the constant k. [2]

$Ne^{kn} = 0.79^n N$
$\therefore 0.79^n = e^{kn}$
$\ln 0.79^n = (kn) \ln e$
$\cancel{n} \ln 0.79 = \cancel{n} k$

$\therefore k = \ln 0.79$
$= \underline{-0.236}$



In the diagram, PQ is the diameter of a circle, centre O. Triangle PQR is an isosceles triangle with $PQ = PR$. The line QR intersects the circle at T. The tangent to the circle at T meets PR at S.

(i) Show that angle $TSR = 90°$. [5]

since PQ is diameter of circle, $\angle QTP = 90°$ (Right-angled Δ in semi-circle)

Let $\angle PQR = \theta$,
then $\angle PRQ = \theta$ (isos. Δ because $PQ = PR$)

$\angle STP = \angle PQR = \theta$ (alt. seg. theorem)

$\angle STR = 180° - 90° - \theta = 90° - \theta$ ($\angle$s on a str. line)

$\angle TSR = 180° - \angle STR - \angle PRQ$
$= 180° - (90° - \theta) - \theta$
$= 90°$ (sum of $\angle$s in $\Delta = 180°$)

(ii) Explain why the circle passing through the points S, R and T has its centre at the midpoint of TR. [2]

Δ SRT is a right-angled Δ,
hence TR is the diameter of the circle passing through S, R and T. (Right-angled Δ in semi-circle)

If TR is the diameter of the circle,
then the centre of TR is the centre of the circle.

7 (i) Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2-\frac{x}{8}\right)^6$. [3]

$$\left(2-\frac{x}{8}\right)^6 = 2^6 + \binom{6}{1}2^5\left(-\frac{x}{8}\right) + \binom{6}{2}2^4\left(-\frac{x}{8}\right)^2 + \cdots$$
$$= 64 - 24x + \frac{15}{4}x^2 + \cdots$$

(ii) In the expansion of $(4+kx+x^2)\left(2-\frac{x}{8}\right)^6$, the sum of the coefficients of x and x^2 is zero. Find the value of the constant k. [4]

$$(4+kx+x^2)\left(2-\frac{x}{8}\right)^6$$
$$= (4+kx+x^2)\left(64-24x+\frac{15}{4}x^2+\cdots\right)$$

Coef. of $x = 4(-24) + 64(k)$
$= -96 + 64k$

Coef. of $x^2 = 4\left(\frac{15}{4}\right) + k(-24) + 64$
$= 79 - 24k$

Given $-96 + 64k + 79 - 24k = 0$
$40k = 17$
$k = \frac{17}{40}$

8 The equation of a curve is $y = x + \frac{2x+5}{x-2}$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

$$\frac{dy}{dx} = 1 + \frac{(x-2)(2) - (2x+5)(1)}{(x-2)^2}$$

$$= 1 + \frac{2x - 4 - 2x - 5}{(x-2)^2}$$

$$= 1 - \frac{9}{(x-2)^2}$$

$$\frac{d^2y}{dx^2} = 18(x-2)^{-3} = \frac{18}{(x-2)^3}$$

(ii) Find the x-coordinate of each of the stationary points of the curve. [3]

Let $\frac{dy}{dx} = 0$

$$1 - \frac{9}{(x-2)^2} = 0$$

$$\frac{9}{(x-2)^2} = 1$$

$$(x-2)^2 = 9$$

$$x - 2 = \pm 3$$

$$x = 2 \pm 3$$

$\therefore x = -1$ or 5

(iii) Find the nature of each stationary point. [2]

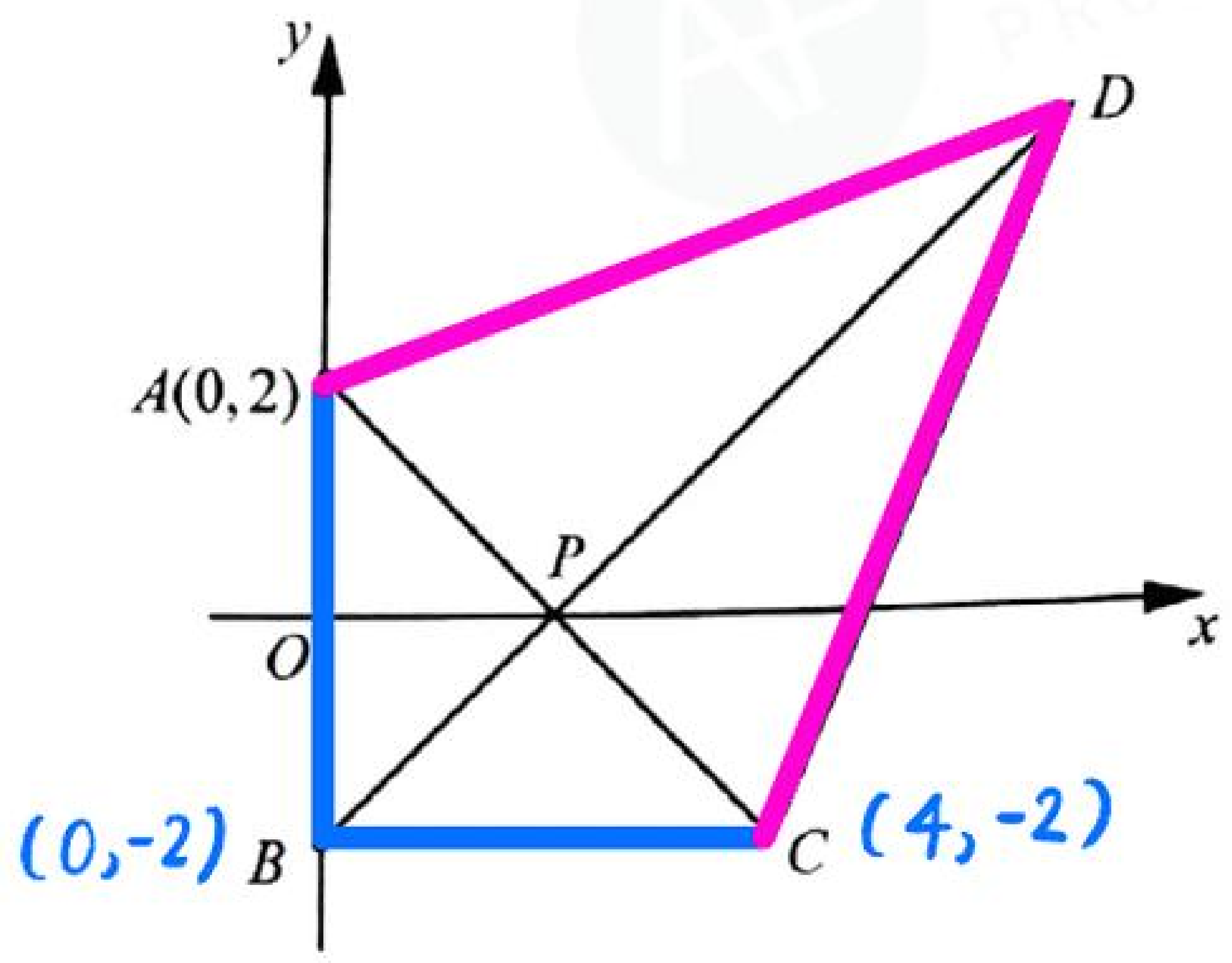
When $x = -1$, $\frac{d^2y}{dx^2} = \frac{18}{(-1-2)^3} = \frac{18}{-27} = -\frac{2}{3} < 0$

(Maximum Point)

When $x = 5$, $\frac{d^2y}{dx^2} = \frac{18}{(5-2)^3} = \frac{18}{27} = \frac{2}{3} > 0$

(Minimum Point)

9



The diagram shows a kite $ABCD$ in which $AB = BC$ and $AD = DC$. The points $A(0, 2)$ and B lie on the y-axis. The diagonals AC and BD intersect at the point P on the x-axis.
Given that the length of AB is 4 units,

(i) explain why BC is parallel to the x-axis, [2]

If the length of AB is 4 units, $B = (0, -2)$
Since O is the midpoint of AB,
and P is the midpoint of AC,
By midpoint theorem, $OP \parallel BC$
i.e. BC is parallel to the x-axis.

(ii) find the coordinates of C. [1]

$\because AB = BC = 4$ units,
$C = (4, -2)$

Given further that the area of the kite is 28 units^2,

(iii) find the coordinates of D. [5]

P is midpoint of $AC = \left(\frac{0+4}{2}, \frac{2-2}{2}\right)$
$= (2, 0)$

Let $D = (a, b)$

gradient BP = gradient BD

$\frac{-2-0}{0-2} = \frac{b-(-2)}{a-0}$

$1 = \frac{b+2}{a}$

$\therefore a = b+2$ ——— (1)

Since area of kite = 28 units^2

$\frac{1}{2}\begin{vmatrix} 0 & 0 & 4 & a & 0 \\ 2 & -2 & -2 & b & 2 \end{vmatrix} = 28$

$|(4b+2a)-(-8-2a)| = 56$

$|4b+4a+8| = 56$

$|b+a+2| = 14$ ——— (2)

Sub (1) into (2):

$|b+b+2+2| = 14$

$2b+4 = \pm 14$

$2b = -4 \pm 14$

$\therefore b = 5$ or -9 (rej.)

When $b=5$, $a=7$

$\therefore$ $D = (7, 5)$

10 **(a)** Find the values of x and y which satisfy the equations

$$3^{x+y} = \sqrt[3]{27},$$
$$\frac{4^y}{2^x} = \left(\frac{1}{2}\right)^{-3}.$$

[4]

$3^{x+y} = 27^{\frac{1}{3}}$

$3^{x+y} = 3$

$\therefore x + y = 1$

$x = 1 - y$ ——— (1)

$\frac{4^y}{2^x} = \left(\frac{1}{2}\right)^{-3}$

$\frac{2^{2y}}{2^x} = 2^3$

$2^{2y-x} = 2^3$

$\therefore 2y - x = 3$

$x = 2y - 3$ ——— (2)

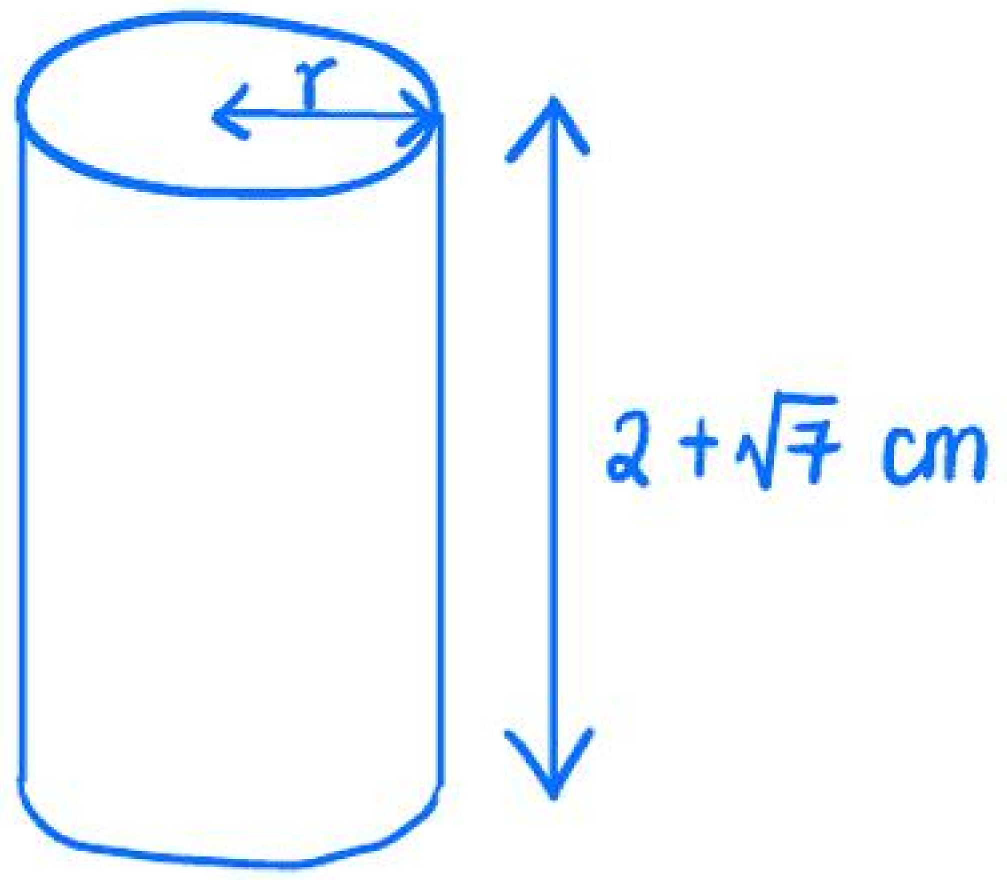
$\Rightarrow 1 - y = 2y - 3$

$3y = 4$

$\underline{y = \frac{4}{3}}$

hence, $\underline{x = 1 - \frac{4}{3} = \frac{-1}{3}}$

(b) A circular cylinder of volume $(3\sqrt{7}-6)\pi$ cm^3 has a height of $(2+\sqrt{7})$ cm and a radius of r cm. Without using a calculator, obtain an expression for r^2 in the form $(a+b\sqrt{7})$, where a and b are integers. [4]



$$V = \pi r^2 h$$

$$(3\sqrt{7}-6)\cancel{\pi} = \cancel{\pi} r^2 (2+\sqrt{7})$$

$$r^2 = \frac{3\sqrt{7}-6}{\sqrt{7}+2}$$

$$= \frac{3\sqrt{7}-6}{\sqrt{7}+2} \cdot \frac{\sqrt{7}-2}{\sqrt{7}-2}$$

$$= \frac{3(7) - 6\sqrt{7} - 6\sqrt{7} + 12}{7-4}$$

$$= \frac{33 - 12\sqrt{7}}{3}$$

$$= \underline{11 - 4\sqrt{7}}$$

11 A dot on a computer screen moves in a straight line so that, t seconds after leaving a fixed point O, its displacement, s cm, from O is modelled by $s = t^3 - 6t^2 + 9t$.

(i) Find the values of t at which the dot is instantaneously at rest. [3]

$s = t^3 - 6t^2 + 9t$

$v = \frac{ds}{dt} = 3t^2 - 12t + 9$

Let $3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

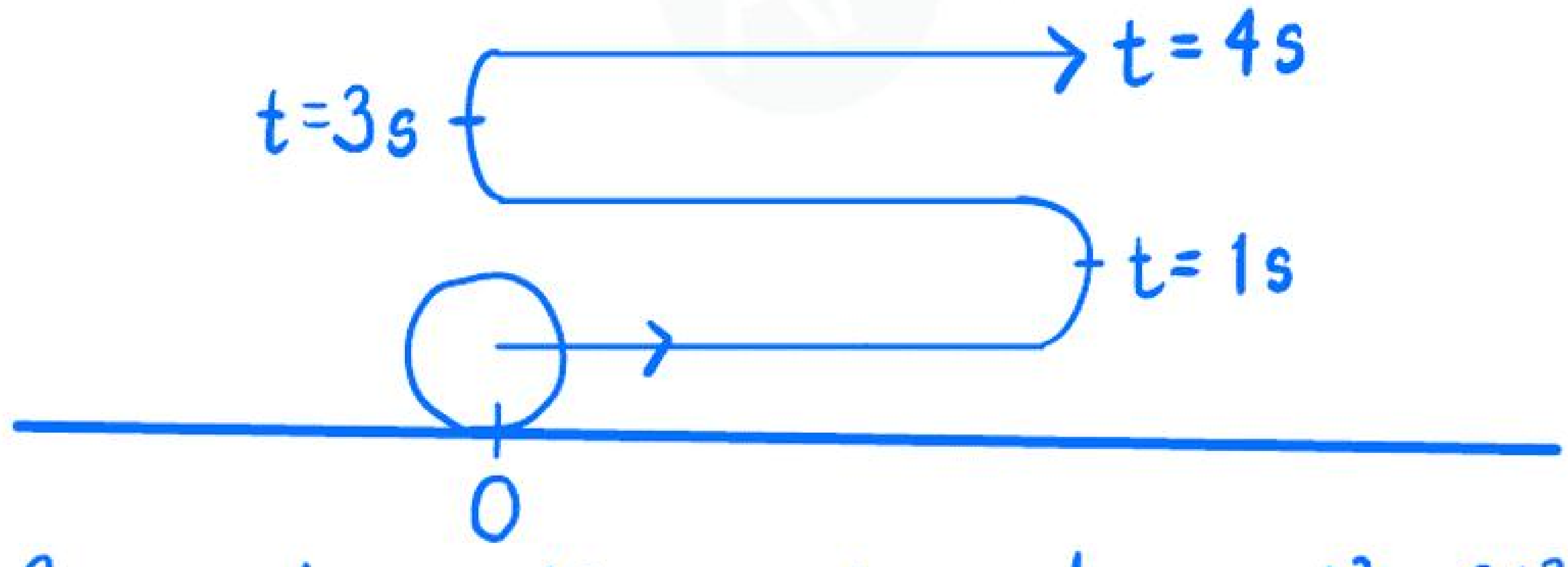
$t = 1s$ or $3s$

(ii) Find the acceleration of the dot when it first comes to instantaneous rest. [2]

$a = \frac{dV}{dt} = 6t - 12$

when $t = 1s$, $a = 6 - 12 = -6cm/s^2$

(iii) Explain clearly why the total distance travelled by the dot in the interval $t = 0$ to $t = 4$ is **not** obtained by finding the value of s when $t = 4$. [2]



By substituting $t = 4s$ into $s = t^3 - 6t^2 + 9t$, we obtained the displacement of the dot from O, not the total distance travelled in the first 4 seconds.

(iv) Find the total distance travelled by the dot in the interval $t = 0$ to $t = 4$. [3]

when $t = 1s$, $s = 1 - 6 + 9 = 4$ cm.

when $t = 3s$, $s = 3^3 - 6(3^2) + 9(3) = 0$ cm.

when $t = 4s$, $s = 4^3 - 6(4^2) + 9(4) = 4$ cm.

Total distance $= 4 + 4 + 4 = \underline{12 \text{ cm}}$.

12 It is given that $f(x) = 2\sin 2x$ and $g(x) = 3\cos\left(\frac{x}{2}\right) - 1$.

(i) State the least and greatest values of $f(x)$. [1]

$f(x)_{least} = \underline{-2}$

$f(x)_{greatest} = \underline{2}$

(ii) State the least and greatest values of $g(x)$. [2]

$g(x)_{least} = \underline{-4}$

$g(x)_{greatest} = \underline{2}$

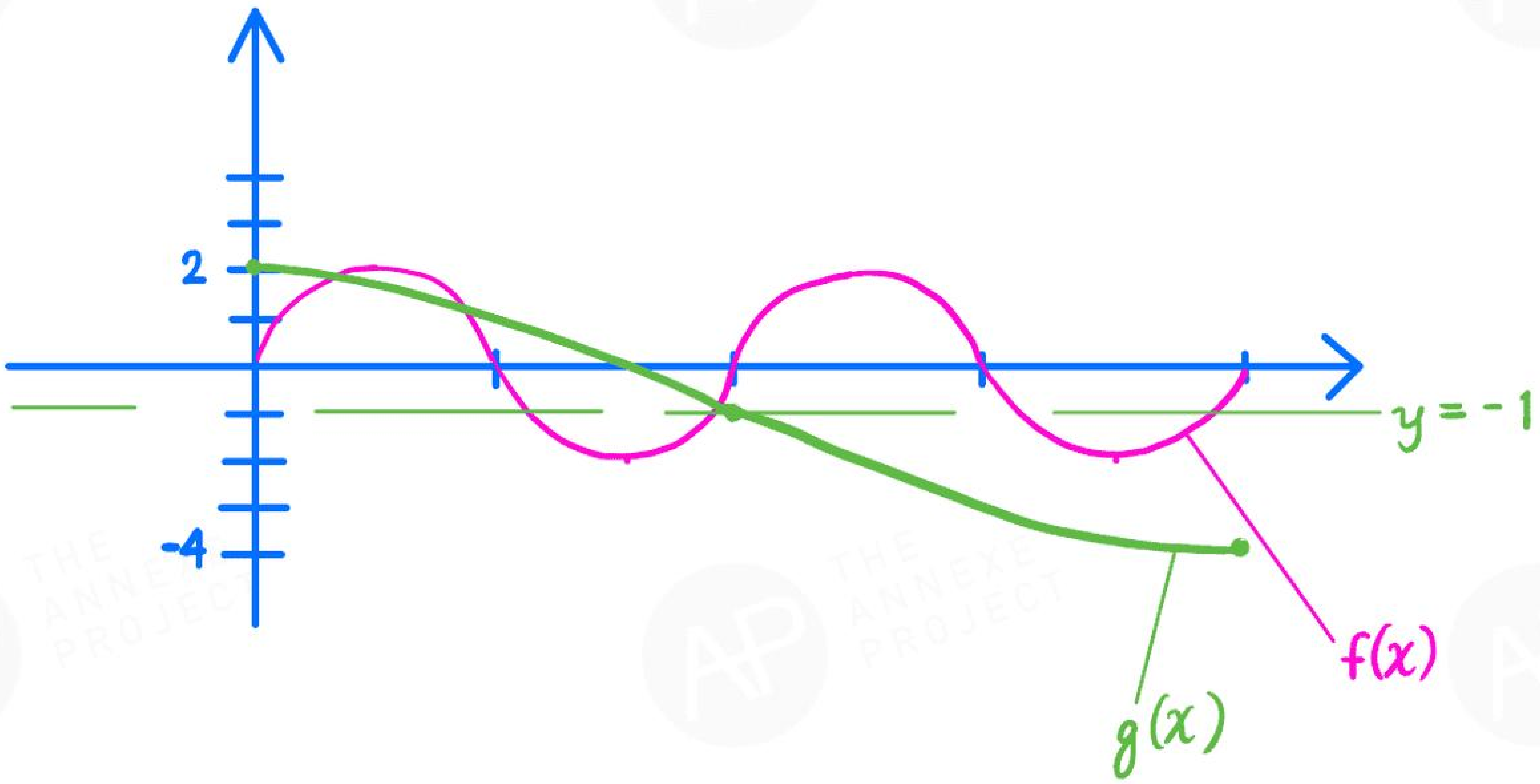
(iii) State the period of $f(x)$. [1]

$\frac{360°}{2} = \underline{180°}$

(iv) State the period of $g(x)$. [1]

$\frac{360°}{\left(\frac{1}{2}\right)} = \underline{720°}$

(v) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0° \leqslant x \leqslant 360°$. [4]



(vi) State the number of solutions of the equation $2\sin 2x + 1 = 3\cos\left(\frac{x}{2}\right)$ for $0° \leqslant x \leqslant 360°$. [1]

From the number of intersections shown above, there are 3 solutions.