MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER	S					INDEX NUMBER					

ADDITIONAL MATHEMATICS

4049/02

Paper 2

For examination from 2021

SPECIMEN PAPER

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment
International Education

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\frac{2x^3-8}{x^3+4x}$ in partial fractions. [6]

Step 1: Note that this is an improper fraction

$$\begin{array}{r|l} & \;2 \\ \hline x^3+4x & 2x^3+0x^2+0x-8 \\ & -(2x^3 \qquad\quad +8x) \\ \hline & \qquad\qquad -8x-8 \\ \hline \end{array}$$

Step 2: $\frac{2x^3-8}{x^3+4x} = 2 - \frac{8x+8}{x(x^2+4)}$

Let $\frac{8x+8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$= \frac{A(x^2+4)+x(Bx+C)}{x(x^2+4)}$

Comparing $8x+8 = A(x^2+4)+x(Bx+C)$

<u>Sub $x=0$:</u> $8 = 4A$

$\therefore A = 2$

<u>Comparing the coefficient of x:</u> $8 = C$

<u>comparing the coefficient of x^2:</u> $0 = A + B$

$0 = 2 + B$

$\therefore B = -2$

Hence, $\frac{2x^3-8}{x^3+4x} = 2 - \left[\frac{2}{x} + \frac{-2x+8}{x^2+4}\right]$

$= \underline{2 - \frac{2}{x} - \frac{8-2x}{x^2+4}}$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

2 **(a)** Variables x and y are related by the equation $yx^n = k$, where n and k are constants. Explain clearly how n and k can be calculated when a graph of $\lg y$ against $\lg x$ is drawn. [3]

$yx^n = K$

$\lg y + n\lg x = \lg K$

$\lg y = -n\lg x + \lg K$

When $\lg Y$ is plotted against $\lg K$, the gradient of the straight line plotted can be equated to $-n$.

The y-intercept can be equated to $\lg K$.

(b) The time for a complete oscillation, t seconds, of a pendulum of length l m is proportional to $\sqrt{l}$. In an experiment with pendulums of different lengths, the following table was obtained.

Length of pendulum, l m	0.2	0.4	0.6	1.0
Time of one oscillation, t sec	0.90	1.27	1.55	2.02

(i) On the grid on page 5, draw a straight line graph to illustrate this data. [2]

(ii) Use your graph to estimate the time of one oscillation for a pendulum of length 0.8 m. [2]

ⓘ $\sqrt{l} \propto t$

$\sqrt{l} = kt$

Plot $\sqrt{l}$ against t.

ⓘⓘ When $l = 0.8$ m,

$\sqrt{l} = 0.894$

from graph, $\underline{t \approx 1.85 s}$

It is known that the correct formula connecting t and l is $t = 2\pi\sqrt{\frac{l}{g}}$, where g is the acceleration due to gravity.

(iii) Use your graph to estimate the value for g. [3]

$t = \frac{2\pi}{\sqrt{g}}\sqrt{l}$

$\sqrt{l} = \frac{\sqrt{g}}{2\pi} t$

from graph, gradient $= 0.48148$

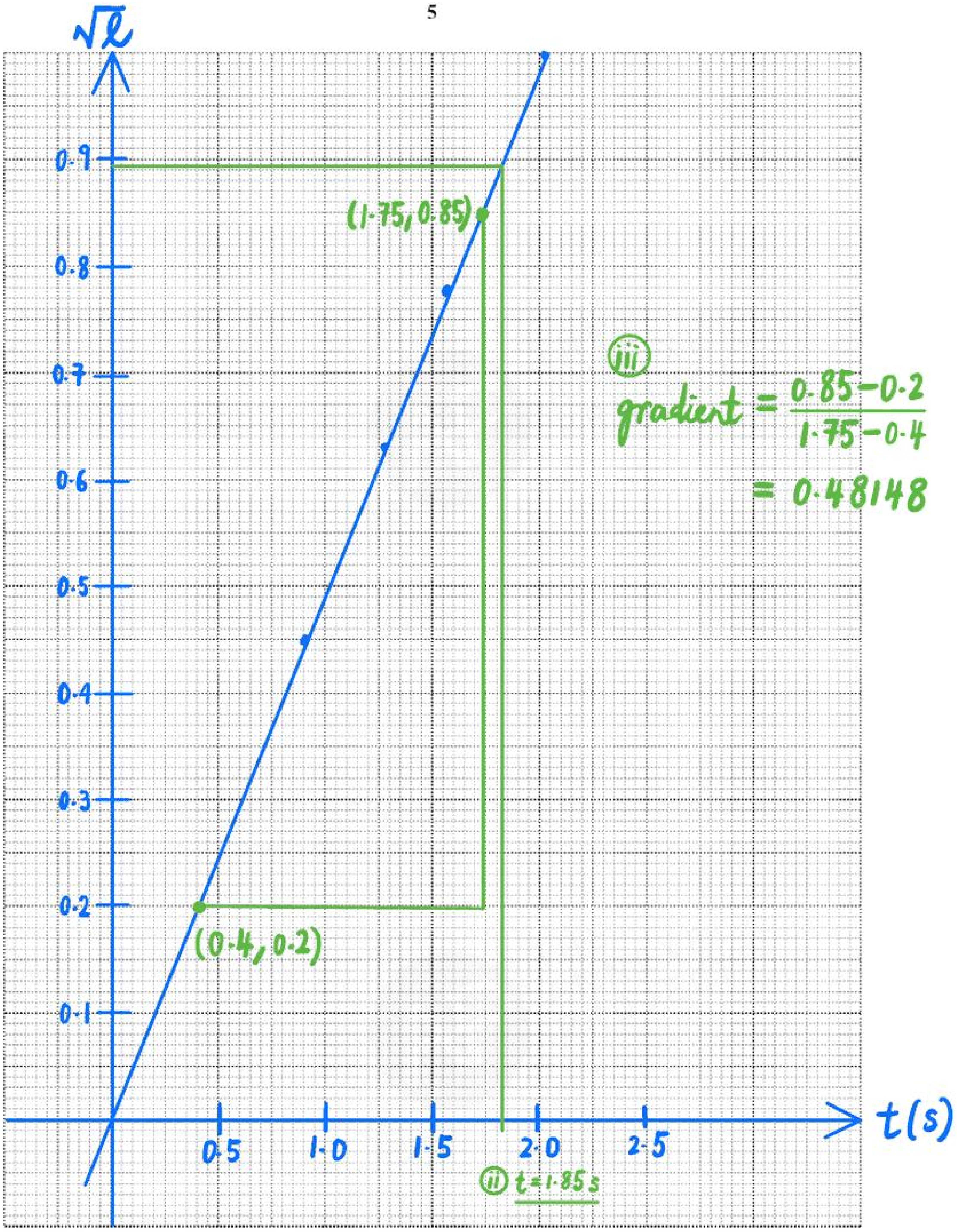
Hence, $\frac{\sqrt{g}}{2\pi} = 0.48148$

$\sqrt{g} = 3.0252$

$\therefore g = \underline{9.15 \text{ m/s}^2}$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

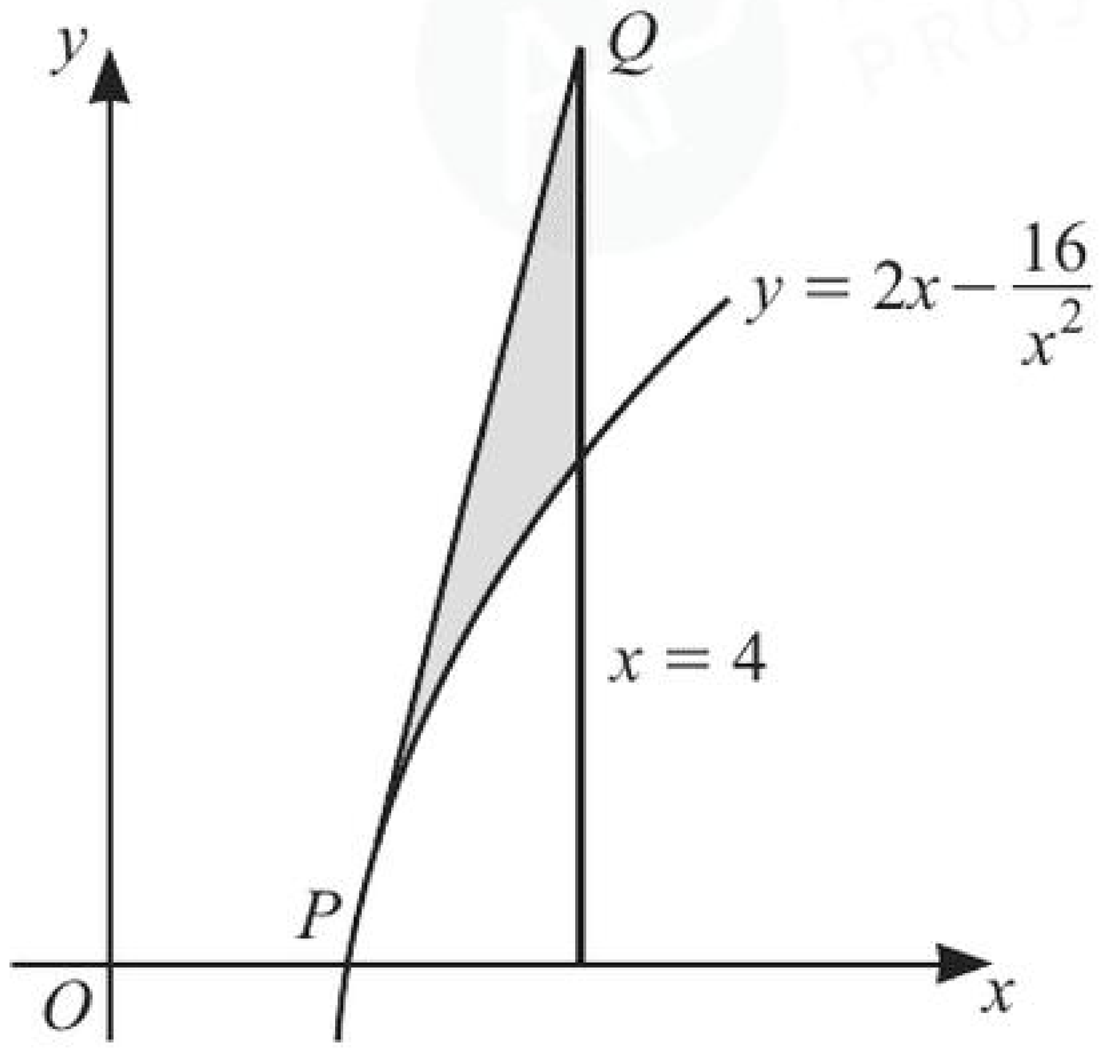
Length of pendulum, l m	0.2	0.4	0.6	1.0
Time of one oscillation, t sec	0.90	1.27	1.55	2.02
$\sqrt{l}$	0.447	0.632	0.775	1.0



(ii) $t = 1.85$ s

(iii)
$$\text{gradient} = \frac{0.85 - 0.2}{1.75 - 0.4}$$
$$= 0.48148$$

3



The diagram shows part of the curve $y = 2x - \frac{16}{x^2}$, which intersects the x-axis at P. The tangent to the curve at P meets the line $x = 4$ at Q.

(a) Find the equation of the line PQ. [5]

let $2x - \frac{16}{x^2} = 0$

$2x^3 - 16 = 0$

$x^3 = 8$

$x = 2$

$\therefore\ P = (2, 0)$

$y = 2x - 16x^{-2}$

$\frac{dy}{dx} = 2 + 32x^{-3}$

When $x = 2$, $\frac{dy}{dx} = 2 + \frac{32}{8} = 6$

$\therefore$ gradient of line PQ $= 6$

Equation of line PQ:

$y - 0 = 6(x - 2)$

$\underline{y = 6x - 12}$

(b) Find the area of the shaded region bounded by the tangent *PQ*, the curve and the line $x = 4$. [5]

When $x = 4$, $y = 6(4) - 12$

$= 12$

$\therefore \quad Q = (4, 12)$

$$\text{Shaded Area} = \text{area of } \triangle - \int_2^4 2x - 16x^{-2}\, dx$$

$$= \left(\tfrac{1}{2} \times 2 \times 12\right) - \left[x^2 - \frac{16x^{-1}}{-1}\right]_2^4$$

$$= 12 - \left[x^2 + \frac{16}{x}\right]_2^4$$

$$= 12 - [(16+4) - (4+8)]$$

$$= 12 - 8$$

$$= \underline{4 \text{ units}^2}$$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

4 **(a)** A curve has equation $y = \frac{2}{x} + k$ and a line has equation $2x + 3y = k$, where k is a constant. Find the set of values of k for which the curve and the line do not intersect and represent this set on a number line. [5]

$y = \frac{2}{x} + k$ —①

$2x + 3y = K$ —②

Sub ① into ②: $2x + 3\left(\frac{2}{x} + k\right) = K$

$2x + \frac{6}{x} + 3K - K = 0$

$2x^2 + 6 + 2Kx = 0$

$2x^2 + 2Kx + 6 = 0$

$b^2 - 4ac < 0$

$(2K)^2 - 4(2)(6) < 0$

$4K^2 - 48 < 0$

$K^2 - 12 < 0$

$K^2 - \sqrt{12}^2 < 0$

$(K - \sqrt{12})(K + \sqrt{12}) < 0$

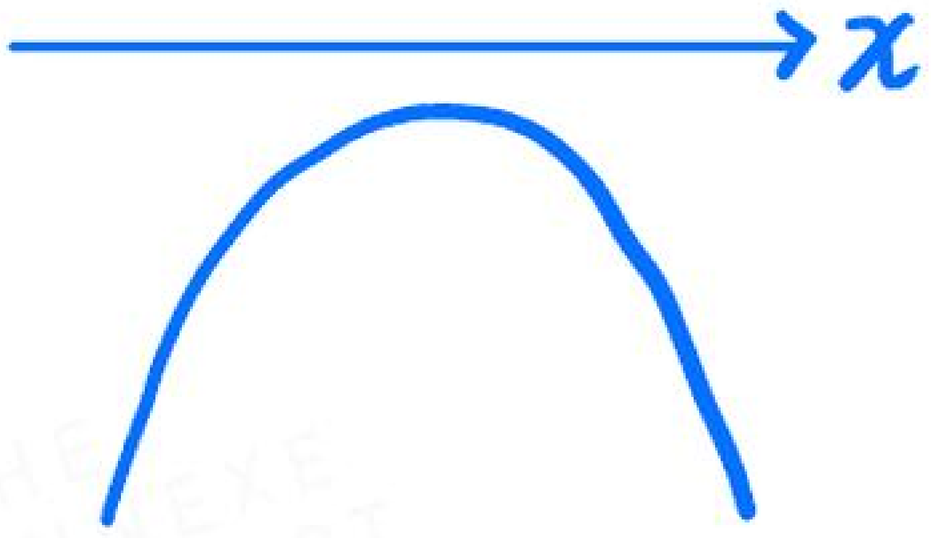
$(K - 2\sqrt{3})(K + 2\sqrt{3}) < 0$

$\therefore$ $-2\sqrt{3} < K < 2\sqrt{3}$

(b) The curve with equation $y = ax^2 + bx + a$, where a and b are constants, lies completely below the x-axis.

(i) Write down the conditions which must apply to a and b. [3]

- firstly, the curve has to be a maximum curve.
  hence, $a < 0$



- secondly, $b^2 - 4(a)(a) < 0$

$b^2 - 4a^2 < 0$

$(b - 2a)(b + 2a) < 0$

$\therefore \ 2a < b < -2a$

(ii) Give an example of possible values for a and b which satisfy the conditions in part **(i)**. [2]

$a = -1, \ b = 1$

5 **(a)** Express $\frac{2x}{2x+3}$ in the form $a+\frac{b}{2x+3}$ where a and b are constants, and hence find $\int \frac{2x}{2x+3}\,dx$. [4]

$$\begin{array}{r} 1 \\ 2x+3 \overline{)\,2x \qquad} \\ -(2x+3) \\ \hline -3 \end{array}$$

$$\therefore \frac{2x}{2x+3} = 1 - \frac{3}{2x+3}$$

Hence, $\int \frac{2x}{2x+3}\,dx = \int 1 - \frac{3}{2x+3}\,dx$

$$= \underline{x - \frac{3}{2}\ln(2x+3) + C}$$

(b) Given that $y = x\ln(2x+3)$, find an expression for $\frac{dy}{dx}$. [3]

$$y = x\ln(2x+3)$$

$$\frac{dy}{dx} = x \cdot \frac{2}{2x+3} + \ln(2x+3)$$

$$= \underline{\frac{2x}{2x+3} + \ln(2x+3)}$$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

(c) Using the results from parts **(a)** and **(b)**, find $\int \ln(2x+3)\,dx$. [3]

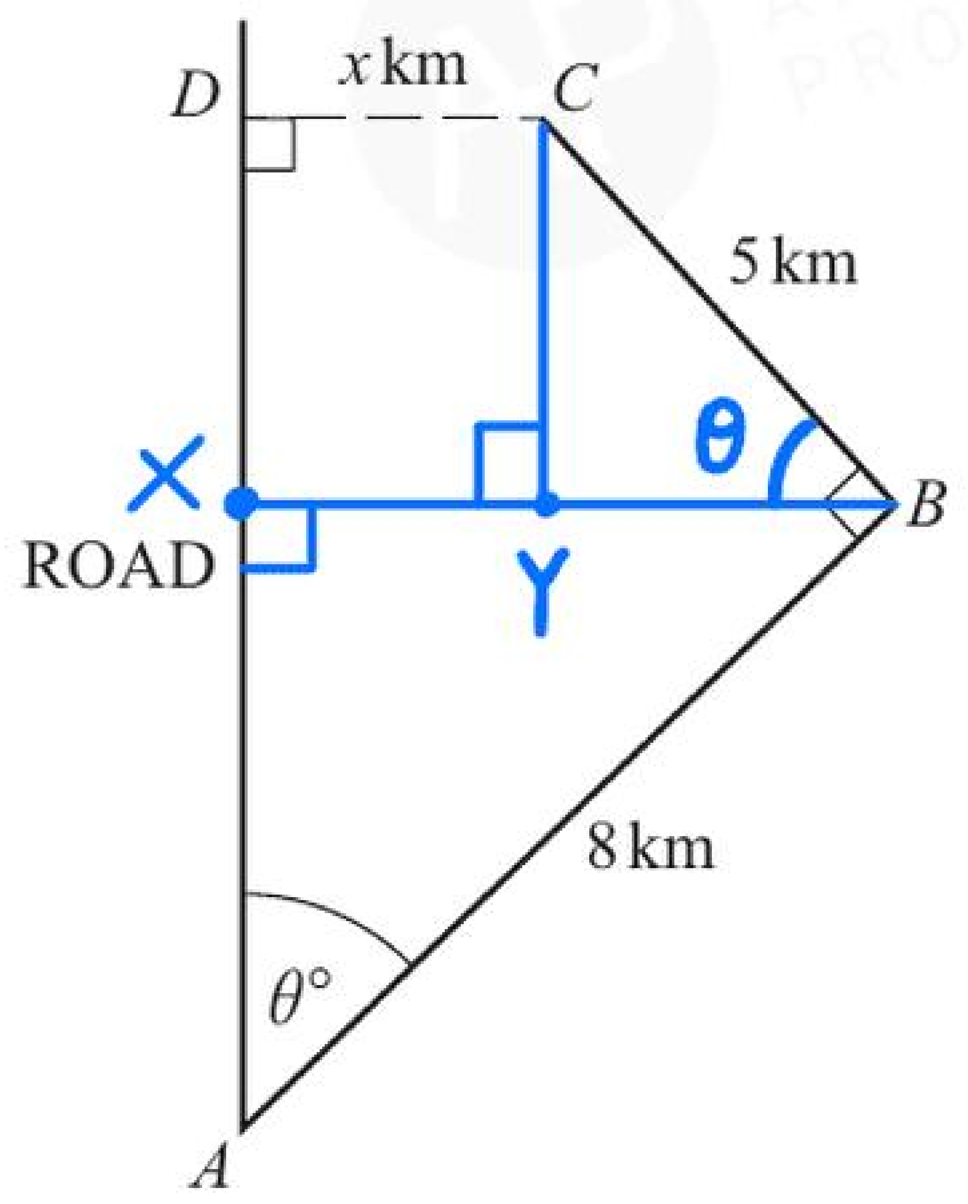
$$\int \ln(2x+3)\,dx$$

$$= \int \frac{2x}{2x+3} + \ln(2x+3) - \frac{2x}{2x+3}\,dx$$

$$= x\ln(2x+3) - x + \frac{3}{2}\ln(2x+3) + C$$

$$= \underline{\left(x + \frac{3}{2}\right)\ln(2x+3) - x + C}$$

6



The diagram shows the path taken by a lady hiker. She walks 8 km in a straight line from a point A to a point B. The path AB is inclined at an acute angle $\theta°$ to a straight road AD. Having reached B, she turns through 90° and walks 5 km to a point C, finding herself x km from the road.

(a) Show that $x = 8\sin\theta - 5\cos\theta$. [3]

$\angle ABX = 180° - 90° - \theta = 90° - \theta$

$\angle CBY = 90° - \angle ABX$

$= 90° - (90° - \theta) = \theta$

$\cos\theta = \frac{BY}{5}$ $\qquad$ $\sin\theta = \frac{BX}{8}$

$\therefore BY = 5\cos\theta$ $\qquad$ $\therefore BX = 8\sin\theta$

$x = BX - BY = \underline{8\sin\theta - 5\cos\theta}$ (shown).

(b) Express $8\sin\theta - 5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° \leqslant \alpha \leqslant 90°$. [4]

$R = \sqrt{8^2 + 5^2} = \sqrt{89}$

$\theta = \tan^{-1}\frac{5}{8} = 32.0°$

$\therefore 8\sin\theta - 5\cos\theta = \underline{\sqrt{89}\sin(\theta - 32.0°)}$

(c) Given that $x = 2$, find the value of θ. [2]

$$2 = \sqrt{89}\ \sin(\theta - 32.0^\circ)$$

$$\theta - 32.0^\circ = \sin^{-1}\frac{2}{\sqrt{89}}$$

$$\underline{\theta = 44.2^\circ}$$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

7 The equation of a curve is $y = e^{x^2 - 4x}$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [5]

$$\frac{dy}{dx} = \underline{e^{x^2-4x} \cdot (2x-4)}$$

$$\frac{d^2y}{dx^2} = e^{x^2-4x} \cdot (2) + (2x-4)e^{x^2-4x} \cdot (2x-4)$$

$$= e^{x^2-4x}\left[2 + (2x-4)^2\right]$$

$$= e^{x^2-4x}(4x^2 - 16x + 18)$$

$$= \underline{2e^{x^2-4x}(2x^2 - 8x + 9)}$$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

(b) Find the exact value of the coordinates of the stationary point. [4]

let $\frac{dy}{dx} = 0$

$e^{x^2-4x} \cdot (2x-4) = 0$

since $e^{x^2-4x} > 0$, $2x = 4$

$x = 2$

When $x = 2$, $y = e^{4-8}$

$= \frac{1}{e^4}$

$\therefore$ coordinates $= \underline{(2, \frac{1}{e^4})}$

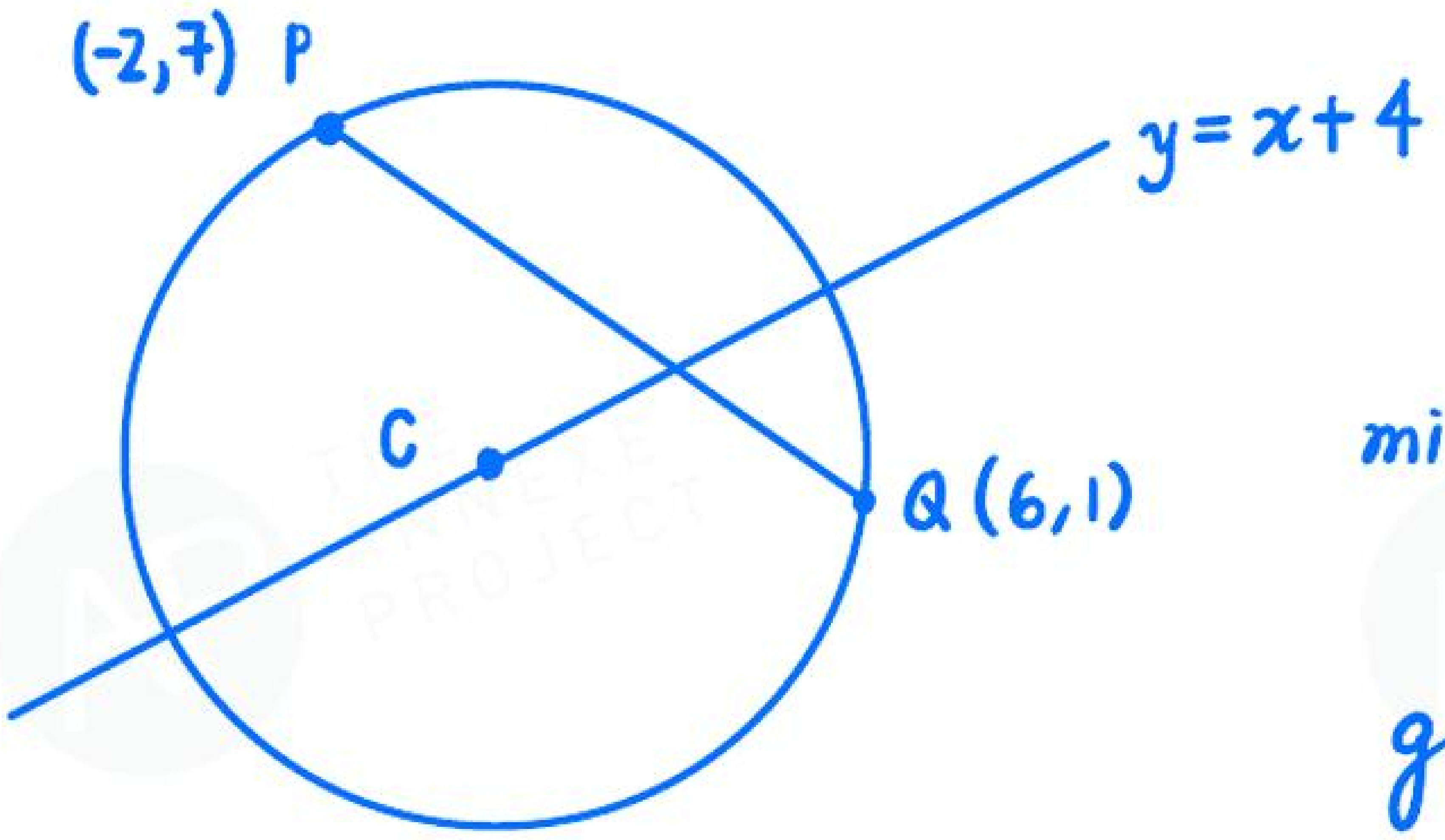
(c) Find the nature of the stationary point. [2]

x	2^-	2	2^+
$\frac{dy}{dx}$	\	—	/

$(2, \frac{1}{e^4})$ is a <u>minimum</u> point.

8 The points P and Q both lie on a circle and have coordinates $(-2, 7)$ and $(6, 1)$ respectively. The centre of the circle lies on the line $y = x + 4$.

(a) Find the equation of the perpendicular bisector of PQ. [5]



midpoint of $PQ = \left(\frac{-2+6}{2}, \frac{7+1}{2}\right)$

$= (2, 4)$

gradient $PQ = \frac{1-7}{6+2} = \frac{-6}{8} = \frac{-3}{4}$

$\therefore$ gradient of perpendicular bisector $= \frac{4}{3}$

Equation of perpendicular bisector of PQ:

$$y - 4 = \frac{4}{3}(x - 2)$$
$$3y - 12 = 4x - 8$$
$$\underline{3y = 4x + 4}$$

(b) Find the equation of the circle. [5]

$y = x + 4$ ①

$3y = 4x + 4$ ②

Sub ① into ②:

$3(x+4) = 4x + 4$

$3x + 12 = 4x + 4$

$\therefore x = 8$

When $x = 8$, $y = 8 + 4$
$= 12$

$\therefore C = (8, 12)$

radius of circle

= length of CP

$= \sqrt{(8+2)^2 + (12-7)^2}$

$= \sqrt{125}$

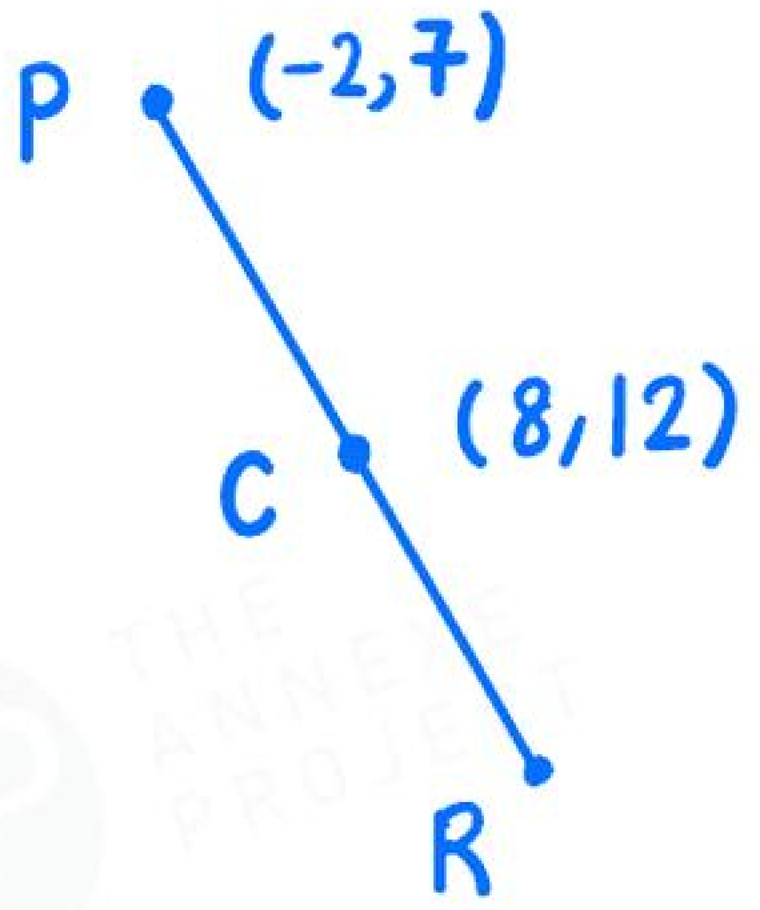
$= 5\sqrt{5}$ units.

equation of circle:

$(x-8)^2 + (y-12)^2 = 125$

The point R is such that PR is a diameter of the circle.

(c) Find the coordinates of R. [2]



Let R be (a, b):

$(8, 12) = \left(\frac{-2+a}{2}, \frac{7+b}{2}\right)$

$8 = \frac{-2+a}{2}$

$16 = -2 + a$

$\therefore a = 18$

$12 = \frac{7+b}{2}$

$24 = 7 + b$

$b = 17$

$R = (18, 17)$

9 **(a)** Use the substitution $u = 2^x$ to solve the equation $2^{2x} - 2^{x+2} = 5$. [4]

$u^2 - 4u - 5 = 0$

$(u+1)(u-5) = 0$

$u = -1$ or $u = 5$

$2^x = -1$ (No Solution)

$2^x = 5$

$x \ln 2 = \ln 5$

$x = \frac{\ln 5}{\ln 2}$

$= \underline{2.32}$

(b) The equation $\log_2 x + \log_8 x = \log_5 25$ has the solution $x = 2^k$. Find the value of k. [4]

$\log_2 x + \frac{\log_2 x}{\log_2 8} = \log_5 5^2$

$\log_2 x + \frac{\log_2 x}{\log_2 2^3} = 2 \log_5 5$

$\log_2 x + \frac{1}{3} \log_2 x = 2$

$\frac{4}{3} \log_2 x = 2$

$\log_2 x = \frac{3}{2}$

$x = 2^{\frac{3}{2}}$

$\therefore \underline{K = \frac{3}{2}}$

(c) Show that the equation $\log_3(4x - 11) - \log_3(x - 3) = 1$ has no real solutions. [4]

$$\log_3 \frac{4x-11}{x-3} = 1$$

$$\frac{4x-11}{x-3} = 3$$

$$4x - 11 = 3x - 9$$

$$x = 2$$

However, if $x = 2$,

then $\log_3(x-3)$ is undefined.

Hence, $\log_3(4x-11) - \log_3(x-3) = 1$ has no solution.

BLANK PAGE