* 0013650219301 *

MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/01

Paper 1

October/November 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **23** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

DC (LK/SW) 199931/4

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!\,(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

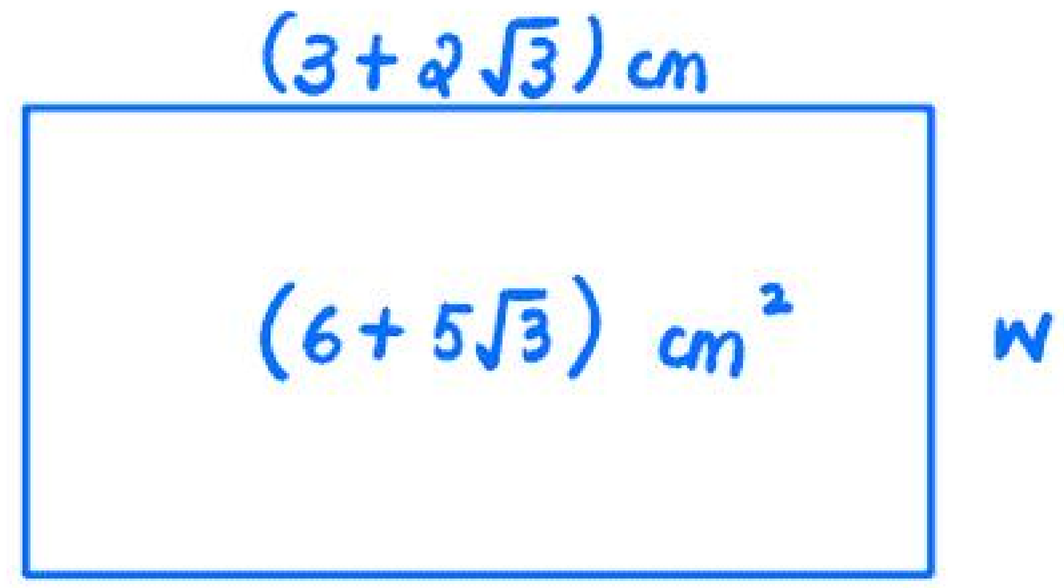
Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 A rectangle has length $(3+2\sqrt{3})$ cm and an area of $(6+5\sqrt{3})\text{cm}^2$. Find, **without using a calculator**, the width of the rectangle, in cm, in the form $(a+b\sqrt{3})$, where a and b are integers. [3]



$$(3+2\sqrt{3})w = 6+5\sqrt{3}$$

$$w = \frac{6+5\sqrt{3}}{3+2\sqrt{3}} \times \frac{3-2\sqrt{3}}{3-2\sqrt{3}}$$

$$= \frac{18-12\sqrt{3}+15\sqrt{3}-30}{9-12}$$

$$= \frac{-12+3\sqrt{3}}{-3}$$

$$= \underline{(4-\sqrt{3})\text{ cm}}$$

* 0013650219304 *

2 The curve $xy+10=0$ and the line $x+2y+1=0$ intersect at the points P and Q. Find the x-coordinate of P and of Q. [3]

$$xy+10=0$$

$$y=\frac{-10}{x} \quad \text{——} \; 1$$

$$x+2y+1=0 \quad \text{——} \; 2$$

Sub 1 into 2:

$$x+2\left(\frac{-10}{x}\right)+1=0$$

$$x^2-20+x=0$$

$$(x-4)(x+5)=0$$

$$\therefore x=-5 \text{ or } 4$$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

3 Express $3-12x-2x^2$ in the form $a(x+b)^2+c$ and hence state the coordinates of the turning point of the curve $y=3-12x-2x^2$. [4]

$$3-12x-2x^2 = -2\left(x^2+6x-\frac{3}{2}\right)$$
$$= -2\left[(x+3)^2-9-\frac{3}{2}\right]$$
$$= -2\left[(x+3)^2-\frac{21}{2}\right]$$
$$= -2(x+3)^2+21$$

where $\underline{a=-2,\ b=3 \text{ and } c=21}$

Maximum point $= \underline{(-3, 21)}$

4 Integrate $\frac{3}{x^2} + \frac{4}{3x-5}$ with respect to x. [4]

$$\int 3x^{-2} + \frac{4}{3x-5}\,dx$$

$$= \frac{3x^{-1}}{-1} + \frac{4\ln(3x-5)}{3} + c$$

$$= \frac{-3}{x} + \frac{4}{3}\ln(3x-5) + C$$ where C is a constant.

5 Express $\frac{13x-6}{x^2(2x-3)}$ in partial fractions. [6]

$$\frac{13x-6}{x^2(2x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-3}$$

$$= \frac{Ax(2x-3) + B(2x-3) + Cx^2}{x^2(2x-3)}$$

hence,

$$13x-6 = Ax(2x-3) + B(2x-3) + Cx^2$$

Sub x as $\frac{3}{2}$: $\frac{27}{2} = \frac{9}{4}C$

$\therefore C = 6$

Sub x as 0: $-6 = -3B$

$\therefore B = 2$

Sub x as 1: $7 = A(-1) + 2(-1) + 6$

$A = -3$

$$\frac{13x-6}{x^2(2x-3)} = \frac{-3}{x} + \frac{2}{x^2} + \frac{6}{2x-3}$$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

* 0013650219308 *

6 A polynomial, P, is $2x^3 - x^2 - 13x + k$, where k is a constant.

(a) Find the value of k given that P leaves a remainder of 6 when divided by $x-2$. [2]

$P(x) = 2x^3 - x^2 - 13x + K$

$P(2) = 16 - 4 - 26 + K = 6$

$\therefore \quad K = 6 + 14$

$= \underline{20}$

(b) In the case where $k = -6$, the quadratic expression $2x^2 + ax - 3$ is a factor of P. Find the value of the constant a. [4]

$P(x) = 2x^3 - x^2 - 13x - 6$

$P(3) = 54 - 9 - 39 - 6$

$= 0$

$\therefore (x-3)$ is a factor.

By Long division,

$P(x) = (x-3)(2x^2+5x+2)$

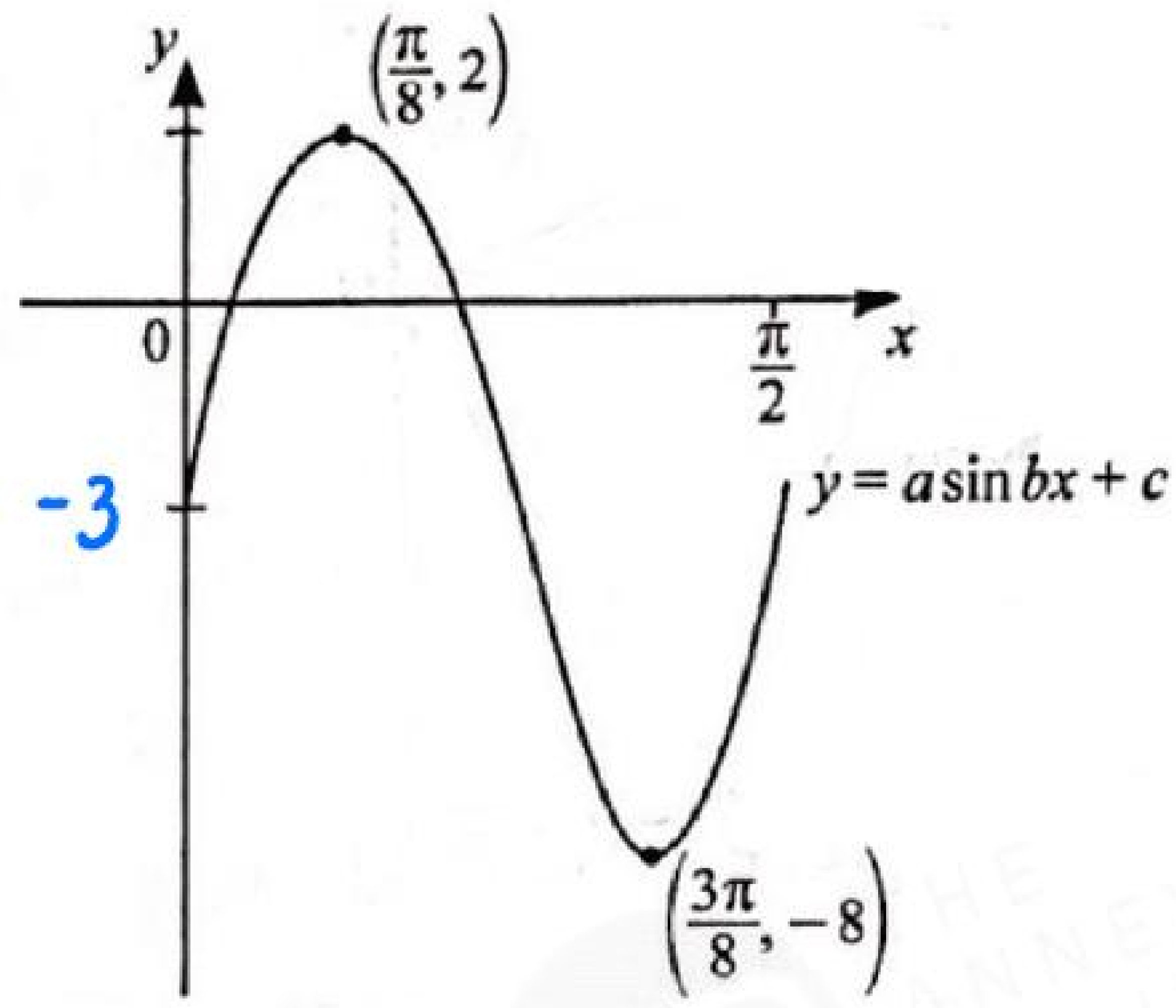
$= (x-3)(2x+1)(x+2)$

$= (2x^2-5x-3)(x+2)$

$$\begin{array}{r|l} & 2x^2 + 5x + 2 \\ \hline x-3 & 2x^3 - x^2 - 13x - 6 \\ & -(2x^3 - 6x^2) \\ \hline & 5x^2 - 13x \\ & -(5x^2 - 15x) \\ \hline & 2x - 6 \\ & -(2x - 6) \\ \hline & 0 \end{array}$$

$\underline{a = -5}$

7



The diagram shows the curve $y = a\sin bx + c$ for $0 \leqslant x \leqslant \frac{\pi}{2}$ radians. The curve has a maximum point at $\left(\frac{\pi}{8}, 2\right)$ and a minimum point at $\left(\frac{3\pi}{8}, -8\right)$.

(a) Explain why $c = -3$. [2]

$$\frac{2 + (-8)}{2} = -3$$

$y = -3$ is the equilibrium line for the above curve where $c = -3$.

(b) Explain why $b = 4$. [2]

Period of sine curve $= \frac{2\pi}{b}$

since period of the above curve $= \frac{\pi}{2}$

$\therefore \frac{\pi}{2} = \frac{2\pi}{b}$

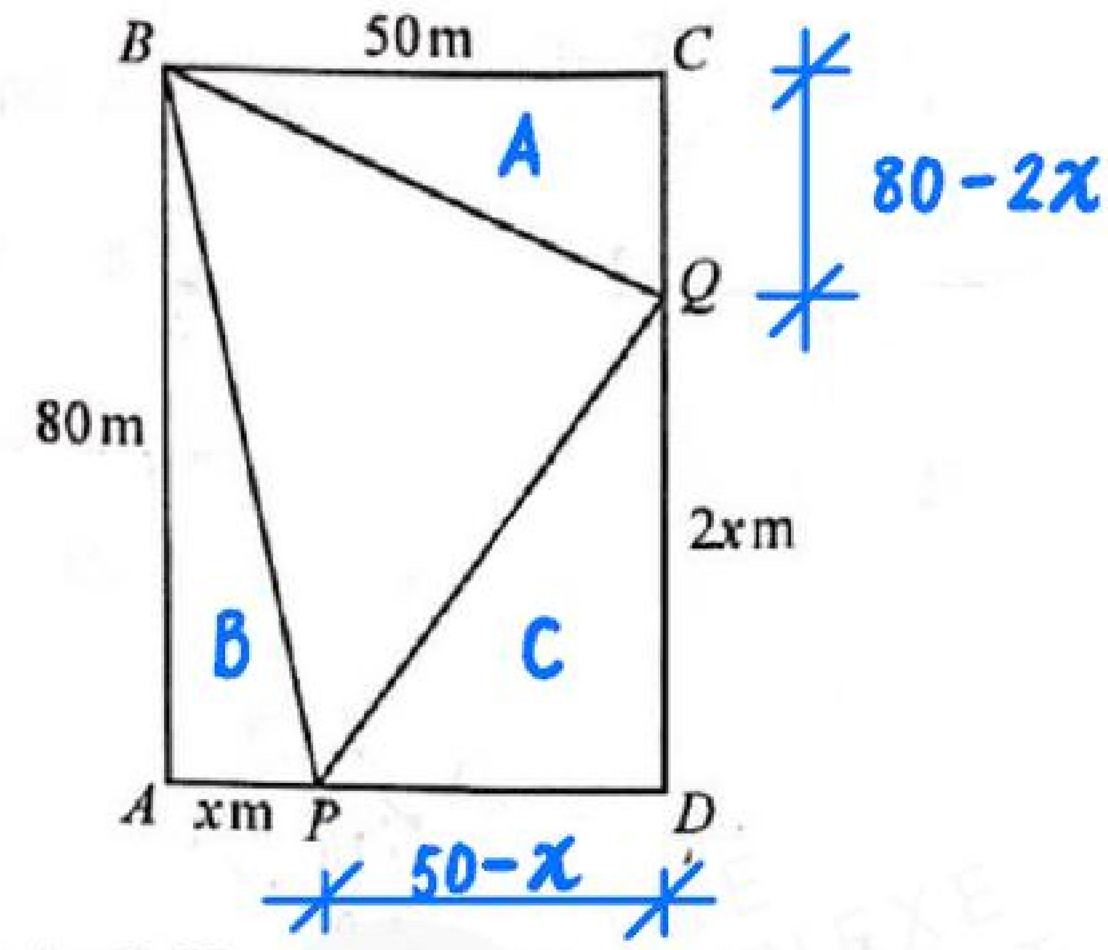
hence, $b = 4$

(c) Hence find the equation of the curve. [2]

Amplitude of curve $= 5$

hence, $\underline{y = 5\sin 4x - 3}$

8



The diagram shows a rectangular field measuring 50 m by 80 m. Points P and Q lie on AD and CD respectively. Paths BP, PQ and QB cross the field. The triangular plot BPQ is to be planted with maize. It is given that $AP = x$ m and $DQ = 2x$ m.

(a) Show that the area, $A\,\text{m}^2$, to be planted with maize is given by

$$A = x^2 - 40x + 2000.$$

[3]

$CQ = (80-2x)$ m and $PD = (50-x)$ m.

A = area of rectangle − area A − area B − area C

$= (80 \times 50) - \frac{1}{2}(50)(80-2x) - \frac{1}{2}(80x) - \frac{1}{2}(2x)(50-x)$

$= 4000 - 2000 + \cancel{50x} - 40x - \cancel{50x} + x^2$

$= \underline{x^2 - 40x + 2000}$ (shown).

(b) Given that x can vary, find the stationary value of A and determine its nature. [4]

$\frac{dA}{dx} = 2x - 40$

let $2x - 40 = 0$

$x = 20$

$\frac{d^2A}{dx^2} = 2$

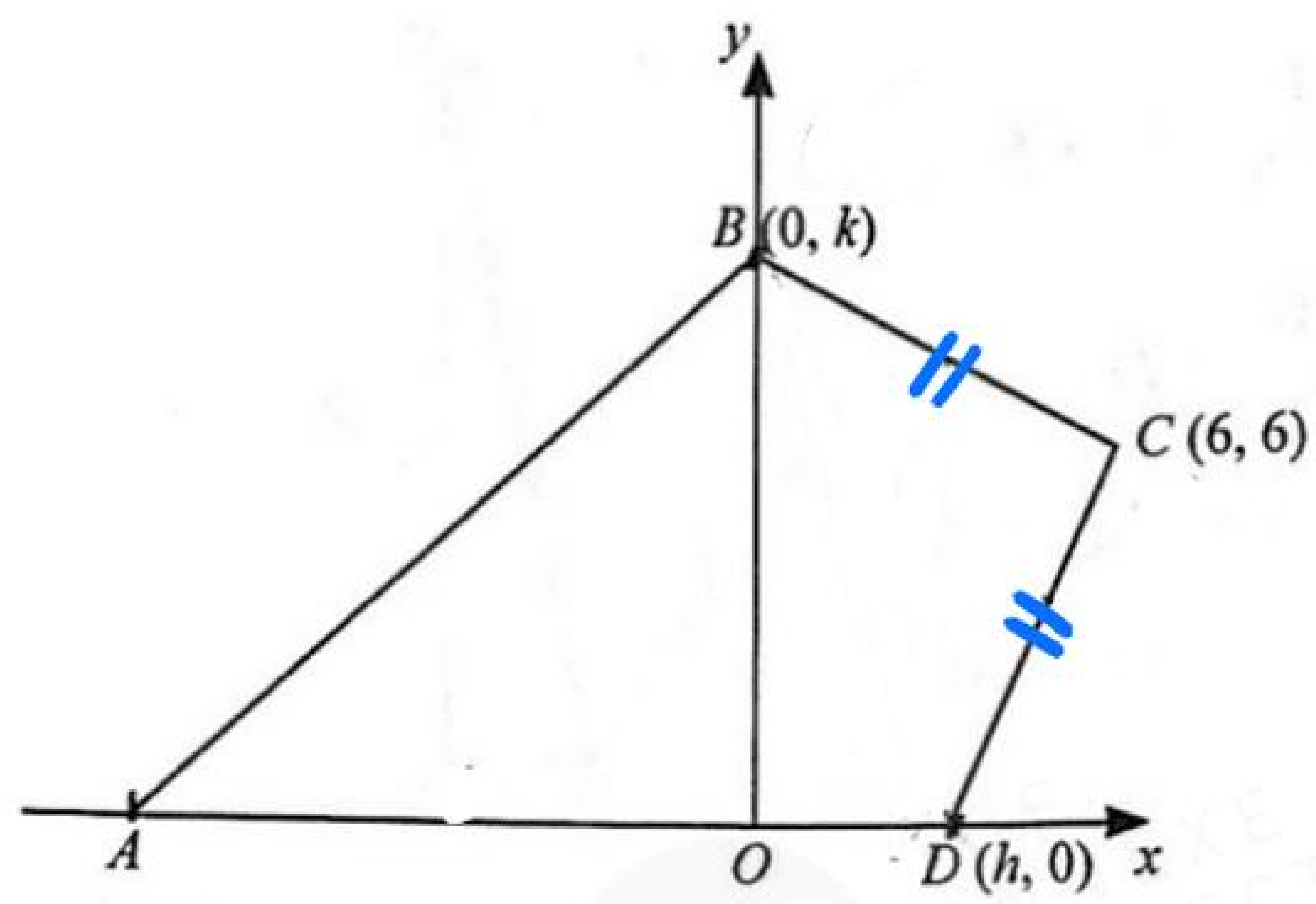
When $x = 20$, $\frac{d^2A}{dx^2} > 0$

hence, A is a minimum.

$A_{min.} = 20^2 - 40(20) + 2000$

$= \underline{1600 \text{ m}^2}$

9



The diagram shows a kite $ABCD$ in which $AB = AD$ and $BC = CD$. Points A and D lie on the x-axis and B lies on the y-axis. Point C is $(6, 6)$, D is $(h, 0)$ and B is $(0, k)$, where h and k are positive constants and $k \neq h$.

(a) Show that $h + k = 12$. [3]

length $BC = \sqrt{(6-0)^2 + (6-k)^2}$

$= \sqrt{36 + 36 - 12k + k^2}$

length $CD = \sqrt{(6-h)^2 + (6-0)^2}$

$= \sqrt{36 - 12h + h^2 + 36}$

since $BC = CD$

$\cancel{72} - 12k + k^2 = \cancel{72} - 12h + h^2$

$k^2 - h^2 - 12k + 12h = 0$

$(k-h)(k+h) - 12(k-h) = 0$

$(k-h)(k+h-12) = 0$

$\therefore k - h = 0$ or $k + h = 12$ (shown).

$k = h$ (rej. because $k \neq h$ given)

It is now given that $h = 4$.

(b) Find the coordinates of A. [4]

$k + 4 = 12$

$\therefore k = 8$

Let A be $(a, 0)$

distance $AB = \sqrt{8^2 + a^2}$

distance $AD = -a + 4$

$AB = AD$

$\therefore \sqrt{64 + a^2} = 4 - a$

$64 + \cancel{a^2} = 16 - 8a + \cancel{a^2}$

$a = -6$

$\therefore \underline{A = (-6, 0)}$

(c) Find the area of the kite. [2]

$$\frac{1}{2}\left|\begin{matrix} -6 & 0 & 6 & 4 & -6 \\ 0 & 8 & 6 & 0 & 0 \end{matrix}\right|$$

$= \frac{1}{2} \left| (-48) - (72) \right|$

$= \frac{1}{2} \left| -120 \right|$

$= \underline{60 \text{ sq. units}}$

10 **(a)** Prove the identity $\frac{\sin\theta}{1-\cos\theta}-\frac{1}{\sin\theta}=\cot\theta$. [4]

$$\text{LHS} = \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} - \frac{1}{\sin\theta}$$

$$= \frac{\sin\theta+\sin\theta\cos\theta}{\sin^2\theta} - \frac{\sin\theta}{\sin^2\theta}$$

$$= \frac{\cancel{\sin\theta}\cos\theta}{\sin^{\cancel{2}}\theta}$$

$$= \cot\theta$$

$$= \text{RHS} \quad (\text{proven})$$

(b) Hence solve the equation $\frac{\sin 2\theta}{1-\cos 2\theta}-\frac{1}{\sin 2\theta}=-2$ for $0° \leqslant \theta \leqslant 180°$. **[4]**

$\cot 2\theta = -2$

$\frac{1}{\tan 2\theta} = -2$

$\tan 2\theta = \frac{-1}{2}$

Step 1: Basic angle for $2\theta = 26.565°$

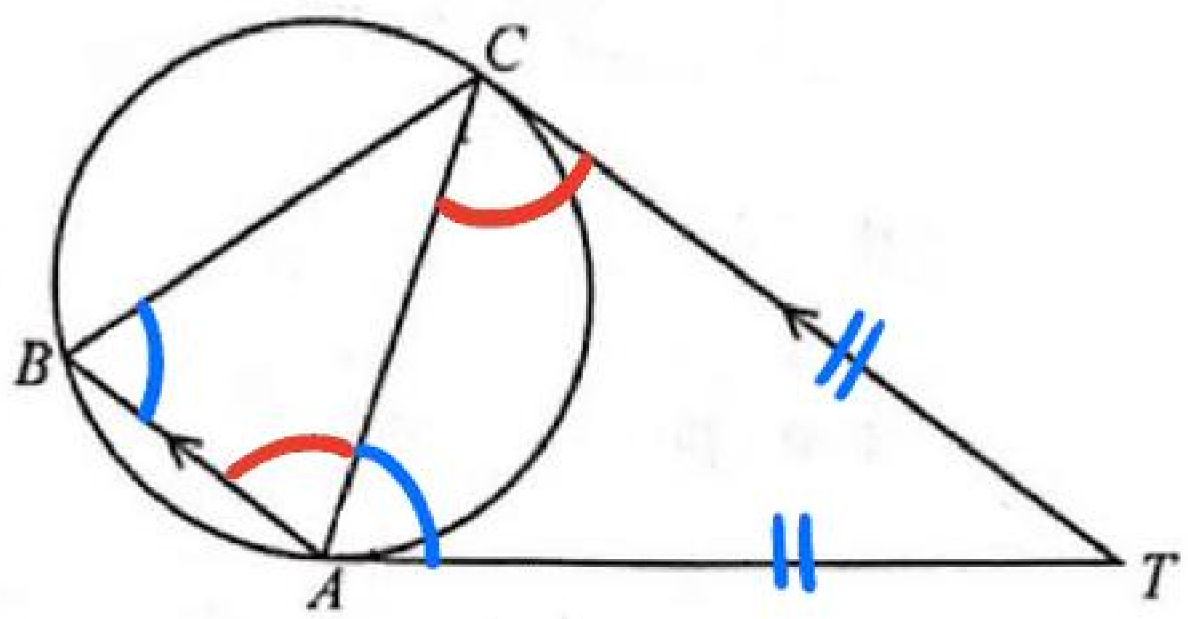
Step 2: 2θ lies in 2^{nd} or 4^{th} quad.

$2\theta = 180° - 26.565°$ or $2\theta = 360° - 26.565°$

$2\theta = 153.43°$ or $2\theta = 333.43°$

Step 3: $\underline{\theta = 76.7° \quad \text{or} \quad 166.7°}$

11



In the diagram, A, B and C lie on a circle. The tangents to the circle at A and C meet at T and BA is parallel to CT.

(a) Prove that triangle ABC is isosceles. [4]

Since CT and AT are tangents to the circle at C and A respectively,

$CT = AT$ (tangent properties)

i.e. $\angle TCA = \angle TAC$

$\angle TAC = \angle CBA$ (tangent chord theorem)

$\angle TCA = \angle CAB$ (alt. $\angle$s, $BA \parallel CT$)

Since $\angle CBA = \angle CAB$,

$\triangle ABC$ is isosceles.

(b) **Prove that the angle BCA = angle CTA.**

Let $\angle TCA = x°$, then $\angle TAC = x°$,

$\angle CBA = x°$

and $\angle CAB = x°$ (from part a).

$\angle BCA = 180° - \angle CBA - \angle CAB$

$= 180° - 2x$ (sum of $\triangle$ is $180°$)

$\angle CTA = 180° - \angle TCA - \angle TAC$

$= 180° - 2x$ (sum of $\triangle$ is $180°$)

$\therefore \angle BCA = \angle CTA$

12 (a) Show that the solution of the equation $6^x = 5 \times 3^{x+1}$ is $x = \dfrac{\lg 15}{\lg 2}$. [4]

$$6^x = 5 \times 3^x \times 3$$

$$\frac{6^x}{3^x} = 15$$

$$2^x = 15$$

$$x \lg 2 = \lg 15$$

$$\therefore \quad x = \frac{\lg 15}{\lg 2}$$

(b) Express the equation $\log_3 x + \log_9(x+2) = 2$ as a cubic equation in x. [4]

$$\log_3 x + \frac{\log_3(x+2)}{\log_3 9} = 2$$

$$\log_3 x + \frac{1}{2}\log_3(x+2) = 2$$

$$\log_3 x\sqrt{x+2} = 2$$

$$x\sqrt{x+2} = 9$$

$$x^2(x+2) = 81$$

$$\therefore x^3 + 2x^2 - 81 = 0$$

13 (a) A spherical balloon is being inflated with helium gas at a constant rate of $500\,\text{cm}^3$ per second. The balloon is initially empty.

(i) Find the radius of the balloon after one minute.
[The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.] [2]

Vol. after 1 min. $= 500 \times 60$
$= 30\,000\ \text{cm}^3$

$\frac{4}{3}\pi r^3 = 30\,000$
$r = 19.276$
$= \underline{19.3\ \text{cm}}$

(ii) Find the rate at which the radius of the balloon is increasing after one minute. [3]

$V = \frac{4}{3}\pi r^3$
$\frac{dV}{dr} = 4\pi r^2$

Given: $\frac{dV}{dt} = 500\ \text{cm}^3/\text{s}$

Find : $\frac{dr}{dt}$ when $t = 60\ \text{s}$, i.e. $r = 19.276\ \text{cm}$

Missing : $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$
$500 = 4\pi(19.276)^2 \cdot \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = 0.10709$
$= \underline{0.107\ \text{cm/s}}$

(b) When a weather balloon is subject to different pressures, the pressure, P atmospheres, and the volume, V litres, of the air in the balloon are related by the formula $PV = k$, where k is a constant. Given that when $P = 1.2$, $V = 2$, find the rate at which V is changing with respect to P when $V = 2$. [4]

Given $PV = k$

$k = 1.2 \times 2 = 2.4$

Hence, $PV = 2.4$

$V = \frac{2.4}{P}$

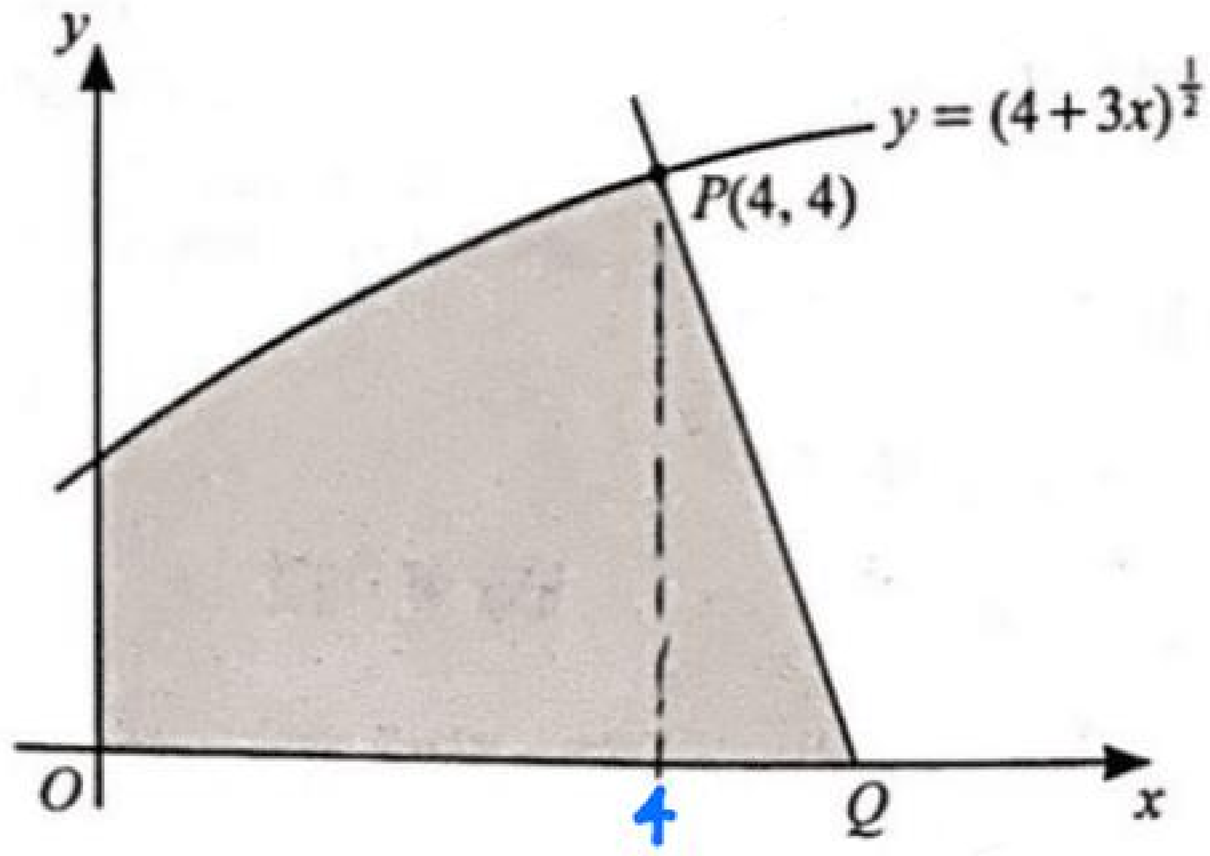
$V = 2.4P^{-1}$

$\frac{dV}{dP} = \frac{-2.4}{P^2}$

since $P = 1.2$ when $V = 2$, $\frac{dV}{dP} = \frac{-2.4}{1.2^2}$

$= \underline{\frac{-5}{3} \ \ell/\text{atmospheres.}}$

14



The diagram shows part of the curve $y=(4+3x)^{\frac{1}{2}}$. The point $P(4, 4)$ lies on the curve and the normal to the curve at P meets the x-axis at Q.

(a) Find the coordinates of Q. [5]

$$y=(4+3x)^{\frac{1}{2}}$$

$$\frac{dy}{dx}=\frac{1}{2}(4+3x)^{-\frac{1}{2}}.(3)$$

$$=\frac{3}{2\sqrt{4+3x}}$$

at P: gradient of tangent $=\frac{3}{2\sqrt{16}}=\frac{3}{8}$

$\therefore$ gradient of normal $=\frac{-8}{3}$

Equation of normal:

$$y-4=\frac{-8}{3}(x-4)$$

$$y=\frac{-8}{3}x+\frac{44}{3}$$

When $y=0$, $x=5.5$

$$Q=(5.5, 0)$$

* 0013650219323 *

(b) Find the area of the shaded region bounded by the curve, the normal PQ and the coordinate axes. [5]

$$A = \int_0^4 (4+3x)^{\frac{1}{2}}\,dx + \frac{1}{2}\left(\frac{3}{2}\right)(4)$$

$$= \left[\frac{(4+3x)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(3)}\right]_0^4 + 3$$

$$= \frac{128}{9} - \frac{16}{9} + 3$$

$$= \frac{139}{9} \text{ or } 15.4 \text{ sq. units}$$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

BLANK PAGE