MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9758/02

Paper 2

For examination from 2025

SPECIMEN PAPER

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

Section A: Pure Mathematics [40 marks]

1 **(a)** The function f is defined as follows.

$$f: x \mapsto x^2 + 6x - 10, \qquad x \in \mathbb{R}$$

(i) By using the method of completing the square, find the range of f. [2]

(ii) Sketch the graph of f. [2]

(b) The function g is defined as follows.

$$g: x \mapsto x^2 + 6x - 10, \qquad x \leqslant \alpha$$

You are given that the function g^{-1} exists.

(i) State the largest possible value of α, explaining why this is the largest value. [2]

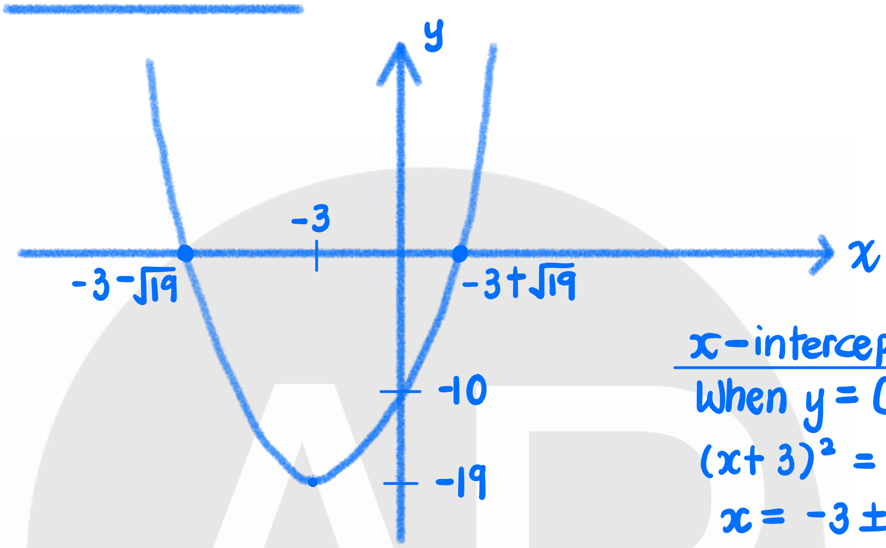
(ii) Find an expression for $g^{-1}(x)$. [2]

(a) (i) $x^2 + 6x - 10 = (x+3)^2 - 9 - 10$

$= (x+3)^2 - 19$

$R_f = [-19, \infty)$

(ii)



x-intercepts:
When $y = 0$
$(x+3)^2 = 19$
$x = -3 \pm \sqrt{19}$

(b) (i) largest value of $\alpha = -3$.

$x = -3$ is the turning point of $g(x)$, it is the largest possible value of α allowing $g(x)$ to be a 1-1 function.

(ii) $y = (x+3)^2 - 19$

$x + 3 = \pm\sqrt{y+19}$

$x = -3 \pm \sqrt{y+19}$

Since $x \leq -3$, $x = -3 - \sqrt{y+19}$

$\therefore g^{-1}(x) = -3 - \sqrt{x+19}, \quad x \geqslant -19$

2 The first four terms of a sequence of numbers are 4, 2, 2 and 4. The sum of the first n terms of this sequence is S_n.

(a) Explain why S_n cannot be a quadratic polynomial in n. [2]

It is given that $S_n = an^3 + bn^2 + cn + d$.

(b) Find the values of a, b, c and d. [4]

(c) Find an expression in terms of n for the nth term of the sequence. [3]

(a). Let $S_n = an^2 + bn + c$.

then S_1: $a + b + c = 4$ —— (1)

S_2: $4a + 2b + c = 6$ —— (2)

S_3: $9a + 3b + c = 8$ —— (3)

By GC : $a = 0$, $b = 2$, $c = 2$

If $a = 0$, S_n is not a quadratic polynomial in n.

(b). Let $S_n = an^3 + bn^2 + cn + d$.

S_1 : $a + b + c + d = 4$ —— (1)

S_2 : $8a + 4b + 2c + d = 6$ —— (2)

S_3 : $27a + 9b + 3c + d = 8$ —— (3)

S_4 : $64a + 16b + 4c + d = 12$ —— (4)

By GC : $a = \frac{1}{3}$, $b = -2$, $c = \frac{17}{3}$, $d = 0$

(c). $T_n = S_n - S_{n-1}$

$= [\frac{1}{3}n^3 - 2n^2 + \frac{17}{3}n] - [\frac{1}{3}(n-1)^3 - 2(n-1)^2 + \frac{17}{3}(n-1)]$

$= \cancel{\frac{1}{3}n^3} - \cancel{2n^2} + \cancel{\frac{17}{3}n} - [\frac{1}{3}(\cancel{n^3} - 3n^2 + 3n - 1) - 2(\cancel{n^2} - 2n + 1) + \cancel{\frac{17}{3}n} - \frac{17}{3}]$

$= n^2 - n + \frac{1}{3} - 4n + 2 + \frac{17}{3}$

$= n^2 - 5n + 8$

3 **(a)** The angle between the vectors $3\mathbf{i} - 2\mathbf{j}$ and $6\mathbf{i} + d\mathbf{j} - \sqrt{7}\,\mathbf{k}$ is $\cos^{-1}\left(\frac{6}{13}\right)$. Find the possible values of d. [3]

Let $\overrightarrow{OA} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 6 \\ d \\ -\sqrt{7} \end{pmatrix}$

$$\theta = \cos^{-1}\left[\frac{\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ d \\ -\sqrt{7} \end{pmatrix}}{\sqrt{13}\,\sqrt{43+d^2}}\right] = \cos^{-1}\frac{6}{13}$$

$$\therefore \quad \frac{18-2d}{\sqrt{13}\,\sqrt{43+d^2}} = \frac{6}{13}$$

$$234 - 26d = 6\sqrt{13}\,\sqrt{43+d^2}$$

$$(234 - 26d)^2 = 36(13)(43+d^2)$$

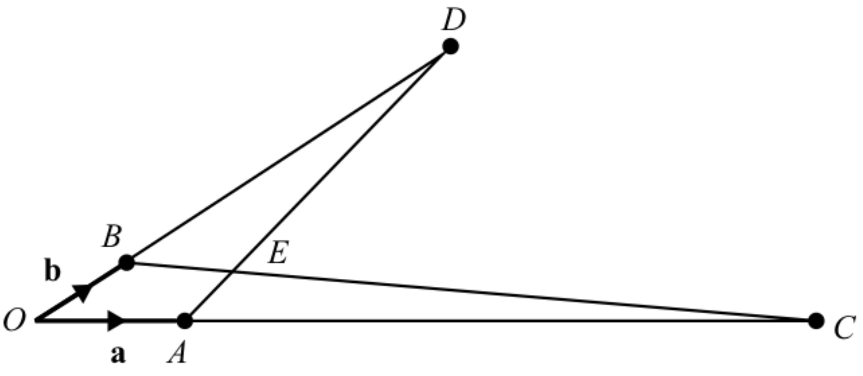
$$54756 - 12168d + 676d^2 = 20124 + 468d^2$$

$$208d^2 - 12168d + 34632 = 0$$

$$2d^2 - 117d + 333 = 0$$

By GC: $d = 3$ or 55.5

(b)

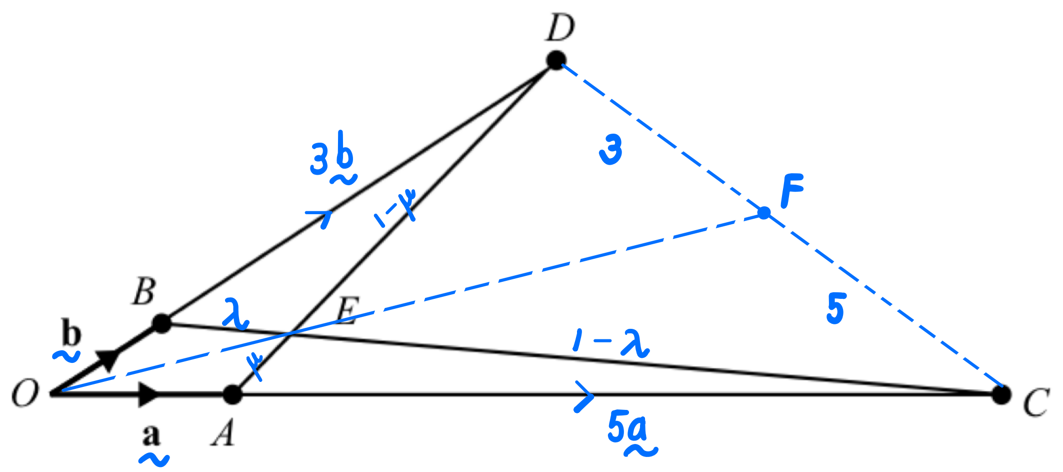


With reference to origin O, the points A, B, C and D are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AC} = 5\mathbf{a}$ and $\overrightarrow{BD} = 3\mathbf{b}$. The lines AD and BC cross at the point E (see diagram).

(i) Find $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [5]

The point F on the line CD is such that $CF : FD = 5 : 3$.

(ii) Show that O, E and F are collinear and find the ratio $OE : OF$. [4]



(i) Given $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$

then $\overrightarrow{OC} = 6\underset{\sim}{a}$ and $\overrightarrow{OD} = 4\underset{\sim}{b}$

By ratio theorem:

$$\overrightarrow{OE} = \lambda\,\overrightarrow{OC} + (1-\lambda)\,\overrightarrow{OB}$$
$$= 6\lambda\,\underset{\sim}{a} + (1-\lambda)\,\underset{\sim}{b} \quad \text{———— (1)}$$

$$\overrightarrow{OE} = \mu\,\overrightarrow{OD} + (1-\mu)\,\overrightarrow{OA}$$
$$= (1-\mu)\underset{\sim}{a} + 4\mu\,\underset{\sim}{b} \quad \text{———— (2)}$$

Comparing coefficients:

$6\lambda = 1-\mu$ $\qquad\qquad$ $1-\lambda = 4\mu$ ———— (4)

$\lambda = \frac{1}{6} - \frac{1}{6}\mu$ ———— (3)

Sub (3) into (4): $1 - \left(\frac{1}{6} - \frac{1}{6}\mu\right) = 4\mu$

$$\frac{5}{6} = \frac{23}{6}\mu$$
$$\therefore \mu = \frac{5}{23}$$
$$\lambda = \frac{1}{6} - \frac{1}{6}\left(\frac{5}{23}\right)$$
$$= \frac{3}{23}$$

Hence, $\overrightarrow{OE} = 6\left(\frac{3}{23}\right)\underset{\sim}{a} + \left(1-\frac{3}{23}\right)\underset{\sim}{b}$

$$= \frac{18}{23}\underset{\sim}{a} + \frac{20}{23}\underset{\sim}{b}$$

(ii) By ratio theorem:

$$\overrightarrow{OF} = \frac{5\,\overrightarrow{OD} + 3\,\overrightarrow{OC}}{8} = \frac{5}{8}(4\underset{\sim}{b}) + \frac{3}{8}(6\underset{\sim}{a}) = \frac{5}{2}\underset{\sim}{b} + \frac{9}{4}\underset{\sim}{a}$$
$$= \frac{23}{8}\left(\frac{18}{23}\underset{\sim}{a} + \frac{20}{23}\underset{\sim}{b}\right)$$
$$= \frac{23}{8}\,\overrightarrow{OE}$$

Since $\overrightarrow{OF} = \frac{23}{8}\overrightarrow{OE}$, and both contain the common point O, then O, E and F are collinear points.

Also, $\overrightarrow{OE} : \overrightarrow{OF} = 8 : 23$

4 **(a)** **(i)** Given that $y = \tan(e^x - 1)$, show that $\frac{dy}{dx} = ke^x(1 + y^2)$, where k is a constant to be found. [2]

(ii) Hence find the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ when $x = 0$. [4]

(b) Write down the first three non-zero terms in the Maclaurin series for $\tan(e^x - 1)$. [1]

(c) The first two non-zero terms in the Maclaurin series for $\tan(e^x - 1)$ are twice as big as the first two non-zero terms in the series expansion of $e^{ax}\ln(1 + nx)$. By using appropriate expansions from the list of formulae, find the constants a and n. Hence find the third non-zero term of the series expansion of $e^{ax}\ln(1 + nx)$ for these values of a and n. [4]

(a) (i)

$y = \tan(e^x - 1)$

$\frac{dy}{dx} = \sec^2(e^x - 1)\cdot(e^x)$

$= e^x[1 + \tan^2(e^x - 1)]$

$= e^x(1 + y^2)$

$\therefore \underline{k = 1}$

(ii)

$\frac{d^2y}{dx^2} = e^x\left(2y\frac{dy}{dx}\right) + (1+y^2)e^x = 2e^x y\frac{dy}{dx} + \frac{dy}{dx}$

$\frac{d^3y}{dx^3} = (2e^x)\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] + \left(y\frac{dy}{dx}\right)(2e^x) + \frac{d^2y}{dx^2}$

<u>When $x = 0$:</u>

$y = \tan 0 = 0$,

$\frac{dy}{dx} = e^0(1+0) = 1$,

$\frac{d^2y}{dx^2} = 2e^0(0)(1) + 1 = 1$,

$\frac{d^3y}{dx^3} = 2e^0[0 + 1] + 0 + 1 = 3$

(b) $\tan(e^x - 1) = x + \frac{1}{2!}x^2 + \frac{3}{3!}x^3 + \cdots$

$= \underline{x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \cdots}$

(c) $e^{ax}\ln(1+nx) = \left[1 + ax + \frac{(ax)^2}{2} + \frac{(ax)^3}{6} + \cdots\right]\left[nx - \frac{(nx)^2}{2} + \frac{(nx)^3}{3} - \cdots\right]$

$= \left(1 + ax + \frac{1}{2}a^2x^2 + \frac{1}{6}a^3x^3 + \cdots\right)\left(nx - \frac{1}{2}n^2x^2 + \frac{1}{3}n^3x^3 + \cdots\right)$

$= nx - \frac{1}{2}n^2x^2 + anx^2 + \frac{1}{3}n^3x^3 - \frac{1}{2}an^2x^3 + \frac{1}{2}a^2nx^3 + \cdots$

Consider:

The first two non-zero terms in the Maclaurin series for $\tan(e^x - 1)$ are twice as big as the first two non-zero terms in the series expansion of $e^{ax}\ln(1 + nx)$.

By comparing coefficients of x and x^2:

$1 = 2n$

$\therefore n = \frac{1}{2}$

$\frac{1}{2} = 2\left(-\frac{1}{2}n^2 + an\right)$

$\frac{1}{2} = -\left(\frac{1}{2}\right)^2 + 2a\left(\frac{1}{2}\right)$

$\frac{1}{2} + \frac{1}{4} = a$

$\therefore a = \frac{3}{4}$

Third non-zero term of $e^{ax}\ln(1+nx)$:

$\left(\frac{1}{3}n^3 - \frac{1}{2}an^2 + \frac{1}{2}a^2n\right)x^3$

$= \left[\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^3 - \frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)^2 + \frac{1}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right)\right]x^3$

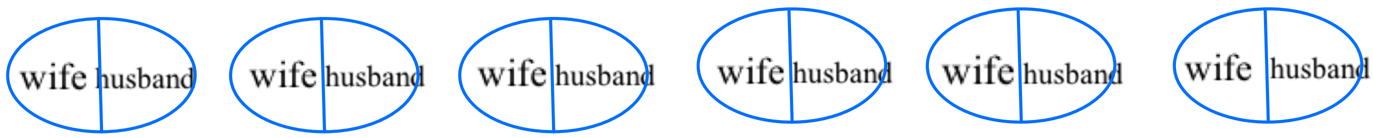
$= \frac{17}{192}x^3$

5 This question is about six couples. Each couple consists of a husband and a wife. The 12 people visit a theatre, and sit in a row of 12 seats.

(a) In how many different ways can the 12 people sit so that each husband and wife in a couple sit next to each other? [2]

(b) In how many ways can the 12 people sit so that the 6 wives all sit next to each other, and exactly 2 of the wives sit next to their husbands. [3]

a

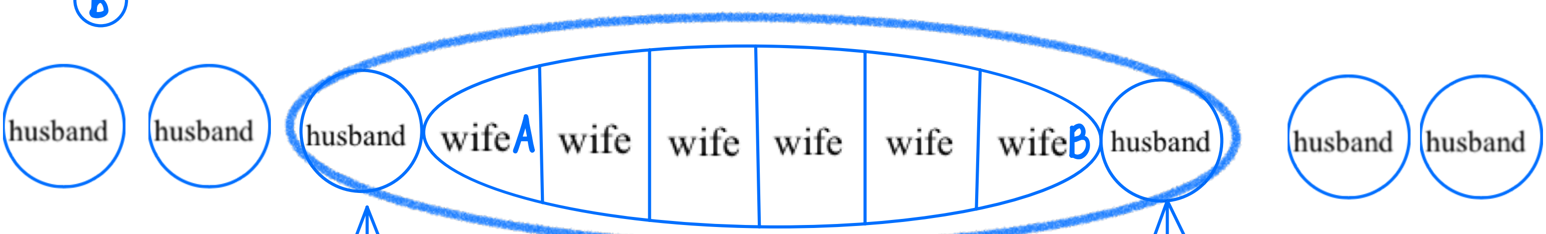


$\underbrace{6!}_{}\times\underbrace{2^6}_{} = \underline{46\,080 \text{ ways}}$

Step I: Arrange the 6 couples in a linear order

Step II: Each couple has 2! ways of arranging themselves within each packet.

b



Step I: 6!
Arrange the 6 wives in a linear order.

<u>1 way only</u>
He has to be the husband of wife A.

<u>1 way only</u>
He has to be the husband of wife B.

Step II: 5!
Arrange the 4 small and 1 big "packets" in a linear order.

total no. of ways = $6! \times 5! = \underline{86\,400 \text{ ways}}$

The group decides to form a committee to arrange future outings. The committee will consist of 3 of the 12 people.

(c) In how many ways can the committee be formed without restriction? [1]

(d) In how many ways can the committee be formed such that at least 1 of the wives will be on the committee but no husband and wife couple will be included? [3]

(c) $^{12}C_3 = \underline{220 \text{ ways}}$

(d) Number of ways where no couples are in the committee − Number of ways only men are in the committee

$$\Big(\underbrace{{}^6C_3}_{\text{choose 3 couples out of the 6.}} \times \underbrace{2^3}_{\text{from each couple, there are 2 ways to choose 1 person}}\Big) - {}^6C_3$$

$= \underline{140 \text{ ways}}$

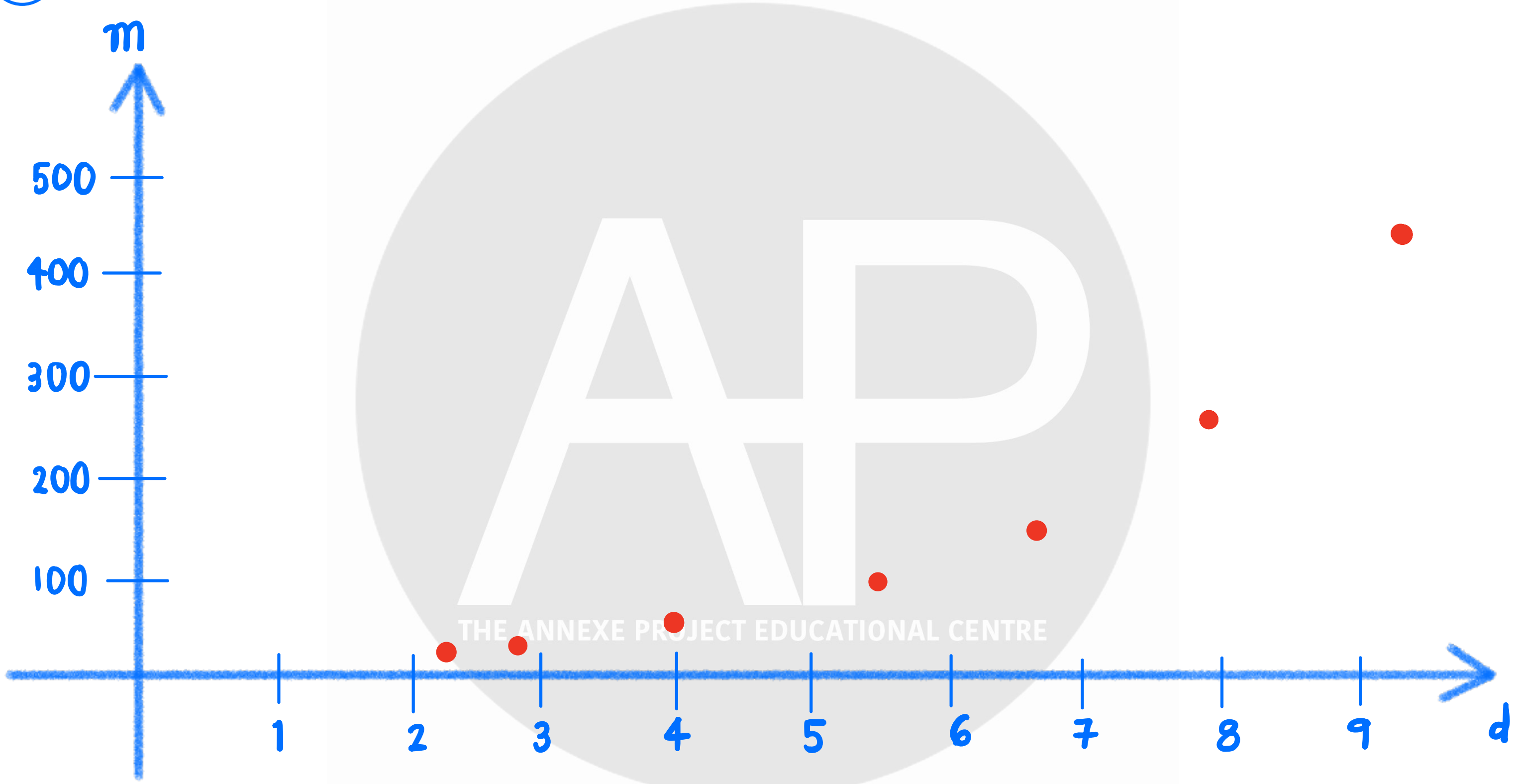
6 Giant pumpkins are often irregular in shape. Growers of giant pumpkins measure the size of a pumpkin using the 'over the top' length. Pumpkin growers keep records so that they can estimate the mass of giant pumpkins while they are still growing. The over the top lengths (d m) and the masses (m kg) of a random sample of 7 giant pumpkins are as follows.

d	2.31	2.90	4.05	5.50	6.70	7.92	9.17
m	11	14	47	104	170	282	449

(a) (i) Draw a scatter diagram of the data. [1]

(ii) Explain how you know from your diagram that the relationship between m and d should not be modelled by a linear equation involving m and d. [1]

a i.



ii. As d increases, m increases at an increasing rate. Hence, m and d do not follow a linear relationship.

(b) Which of the two equations $m = ed^2 + f$ and $m = gd^3 + h$, where e, f, g and h are constants, is the better model for the relationship between m and d? Explain fully how you decided and find the constants for the better equation. [5]

(c) **(i)** Use the equation you chose from part **(b)** to estimate the mass of a giant pumpkin with

(A) over the top length 6 m, [1]

(B) over the top length 12 m. [1]

(ii) Explain which of your two estimates is more reliable. [1]

(b) For $m = ed^2 + f$: $r = 0.9888900551$

For $m = gd^3 + h$: $r = 0.999514626$

Since r value ≈ 1 for $m = gd^3 + h$, this is the better model.

From GC: $m = 0.57165d^3 + 3.7431$

$= 0.572d^3 + 3.74$

(c) i. (A). When $d = 6$ m, $m = 127.22 = 127$ kg

(B). When $d = 12$ m, $m = 991.55 = 992$ kg

ii. (A) is more reliable as interpolation was practised. $d = 6$ m is within the range of data collected.

7 'Crunchers' are sweets that are sold in packets of 8. Each packet is made up of randomly chosen coloured sweets. On average 15% of Crunchers are yellow.

(a) Find the probability that a randomly chosen packet of Crunchers contains no more than one yellow sweet. [2]

(b) Kev buys 70 randomly chosen packets of Crunchers. Find the probability that at least 50 of these packets contain no more than one yellow sweet. [3]

On average the proportion of Crunchers that are red is p. It is known that the modal number of red sweets in a packet is 3.

(c) Use this information to find exactly the range of values that p can take. [4]

(a) Let X be the discrete r.v. denoting no. of yellow sweets in a packet of 8.

$$X \sim B(8, 0.15)$$

$$P(X \leq 1) = 0.65718$$
$$= \underline{0.657}$$

(b) Let Y be no. of packets of Crunchers out of 70 that contain no more than 1 yellow sweet.

$$Y \sim B(70, 0.65718)$$

$$P(Y \geq 50) = 1 - P(Y \leq 49)$$
$$= 0.19010$$
$$= \underline{0.190}$$

(c) Let W be the discrete r.v. denoting no. of red sweets in a packet of 8.

$$W \sim B(8, p)$$

If the mode is 3, then $P(W=3) > P(W=2)$
and $P(W=3) > P(W=4)$

<u>Consider $P(W=3) > P(W=2)$:</u>

$$\binom{8}{3} p^3 (1-p)^5 > \binom{8}{2} p^2 (1-p)^6$$
$$56\, p^{\cancel{3}} \cancel{(1-p)^5} > 28\, \cancel{p^2} (1-p)^{\cancel{6}}$$
$$56p > 28 - 28p$$
$$84p > 28$$
$$p > \frac{1}{3}$$

Consider $P(W=3) > P(W=4)$: $\binom{8}{3} p^3(1-p)^5 > \binom{8}{4} p^4(1-p)^4$

$$56\cancel{p^3}(1-p)^{\cancel{5}} > 70\,p^{\cancel{4}}\,\cancel{(1-p)^4}$$

$$56 - 56p > 70p$$

$$126p < 56$$

$$p < \frac{4}{9}$$

Hence, $\frac{1}{3} < p < \frac{4}{9}$

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8 A bag contains 3 blue discs, 2 red discs and y yellow discs. Li chooses 3 discs at random from the bag, without replacement.

(a) Show that the probability Li chooses 1 blue disc, 1 red disc and 1 yellow disc is $\dfrac{36y}{(y+5)(y+4)(y+3)}$. [1]

Li's discs are replaced in the bag.

Darvina chooses 3 discs at random from the bag, without replacement. The random variable S is the sum of the number of blue discs chosen and twice the number of red discs chosen.

(b) Find an expression in terms of y for $\mathrm{P}(S = 3)$. [2]

(c) Given that $\mathrm{P}(S = 3) = \frac{7}{20}$, calculate the value of y. Hence find the probability distribution of S. [6]

(a) $\dfrac{3}{(y+5)} \times \dfrac{2}{(y+4)} \times \dfrac{y}{(y+3)} \times 3! = \underline{\dfrac{36y}{(y+5)(y+4)(y+3)}}$

(b) $P(S=3) = P(B,B,B) + P(B,R,Y)$

$= \dfrac{3}{(y+5)} \times \dfrac{2}{(y+4)} \times \dfrac{1}{(y+3)} + \dfrac{3}{(y+5)} \times \dfrac{2}{(y+4)} \times \dfrac{y}{(y+3)} \times 3!$

$= \dfrac{6 + 36y}{(y+5)(y+4)(y+3)}$

$= \underline{\dfrac{6(6y+1)}{(y+5)(y+4)(y+3)}}$

(c) $\dfrac{6(6y+1)}{(y+5)(y+4)(y+3)} = \dfrac{7}{20}$

By GC : $\underline{y=1}$ or 2.73 (rej.)

$P(S=2) = P(B,B,Y)$

$= \dfrac{3}{6} \times \dfrac{2}{5} \times \dfrac{1}{4} \times \dfrac{3!}{2!} = 0.15$

$P(S=4) = P(B,B,R) + P(R,R,Y)$

$= \left(\dfrac{3}{6} \times \dfrac{2}{5} \times \dfrac{2}{4} \times \dfrac{3!}{2!}\right) + \left(\dfrac{2}{6} \times \dfrac{1}{5} \times \dfrac{1}{4} \times \dfrac{3!}{2!}\right) = 0.35$

$P(S=5) = P(B,R,R)$

$= \dfrac{3}{6} \times \dfrac{2}{5} \times \dfrac{1}{4} \times \dfrac{3!}{2!} = 0.15$

s	2	3	4	5
$P(S=s)$	0.15	$\frac{7}{20}$	0.35	0.15

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9 In this question you should state the parameters of any normal distribution you use.

A type of metal bolt is manufactured with a nominal radius of 0.8 cm. In fact, the radii of the bolts, measured in cm, have the distribution $N(0.8, 0.01^2)$.

(a) Find the percentage of bolts that have a radius between 0.79 cm and 0.82 cm. [1]

Metal washers are manufactured to fit on the bolts. The inside radii of the washers, measured in cm, have the distribution $N(0.81, 0.012^2)$.

(b) Write down the distribution of the inside circumference of the washers, in cm, and find the circumference that is exceeded by 5% of the washers. [4]

(a) Let X be the r.v. denoting the radius of a bolt.

$$X \sim N(0.8, 0.01^2)$$

$$P(0.79 < X < 0.82) = 0.81859$$
$$= \underline{0.819}$$

<u>81.9%</u> of bolts have a radius between 0.79 cm and 0.82 cm.

(b) Let Y be the r.v. denoting the radius of a washer.

$$Y \sim N(0.81, 0.012^2)$$

Let C be the r.v. denoting the inside circumference of a washer.

$$C = 2\pi Y$$

$\therefore E(C) = 2\pi E(Y)$, $Var(C) = (2\pi)^2 Var(Y)$

$= 2\pi(0.81)$ $\quad = 0.0056849$

$= 5.0894$

$$C \sim N(5.0894, 0.0056849)$$

Let a cm be the circumference exceeded by 5% of the washers:

$$P(C \leq a) = 0.95$$
$$a = 5.2134 = \underline{5.21 \text{ cm}}$$

A bolt and a washer are a 'good fit' if

- the inside radius of the washer is greater than the radius of the bolt and
- the inside radius of the washer is not more than 0.04 cm greater than the radius of the bolt.

(c) A washer is chosen at random and a bolt is chosen at random. Find the probability that the washer and bolt are a good fit. [3]

The outside radii of the washers, measured in cm, have the distribution $N(\mu, \sigma^2)$. It is known that 15% of the washers have an outside radius greater than 1.25 cm and 25% have an outside radius of less than 1.15 cm.

(d) Find the values of μ and σ. [3]

(c)

$$E(Y-X) = E(Y) - E(X)$$
$$= 0.81 - 0.8 = 0.01$$

$$Var(Y-X) = Var(Y) + Var(X)$$
$$= 0.012^2 + 0.01^2 = 0.000244$$

$$Y - X \sim N(0.01, 0.000244)$$

$$\therefore P(0 < Y-X \leq 0.04) = 0.71158$$
$$= \underline{0.712}$$

(d) Let W be the r.v. denoting the outside radius of a washer.

$$W \sim N(\mu, \sigma^2)$$

$P(W > 1.25) = 0.15$

$P\left(Z > \frac{1.25-\mu}{\sigma}\right) = 0.15$

$\therefore P\left(Z \leq \frac{1.25-\mu}{\sigma}\right) = 0.85$

$\frac{1.25-\mu}{\sigma} = 1.0364$

$\mu = 1.25 - 1.0364\sigma$ — (1)

$P(W < 1.15) = 0.25$

$P\left(Z < \frac{1.15-\mu}{\sigma}\right) = 0.25$

$\frac{1.15-\mu}{\sigma} = -0.67449$

$\mu = 1.15 + 0.67449\sigma$ — (2)

Solving (1) and (2): $\underline{\mu = 1.1894 = 1.19}$

$\underline{\sigma = 0.058448 = 0.0584}$

10 The average time required for the manufacture of a certain type of electronic control panel is 17 hours. An alternative manufacturing process is trialled, and the time taken, t hours, for the manufacture of each of 50 randomly chosen control panels using the alternative process is recorded. The results are summarised as follows.

$$n = 50 \qquad \Sigma t = 835.7 \qquad \Sigma t^2 = 14\,067.17$$

The Production Manager wishes to test whether the average time taken for the manufacture of a control panel is different using the alternative process, by carrying out a hypothesis test.

(a) Explain whether the Production Manager should use a 1-tail test or a 2-tail test. [1]

(b) Explain why the Production Manager is able to carry out a hypothesis test without knowing anything about the distribution of the times taken to manufacture the control panels. [1]

(c) **(i)** Find unbiased estimates of the population mean and variance. [2]

(ii) Carry out the test at the 10% level of significance for the Production Manager. Define any symbols you use. [5]

(a) 2-tail test, since the Production Manager wishes to test the difference in the average time and not the increase or decrease in the average time.

(b) sample size is 50. Hence, Central Limit Theorem is used to approximate the distribution to be normal.

(c) i. $\bar{t} = \frac{835.7}{50} = 16.714 = 16.7$

$\sigma^2 = \frac{1}{49}\left[14067.17 - \frac{835.7^2}{50}\right] = 2.0261 = 2.03$

ii. To test $H_0: \mu = 17$ against

$H_1: \mu \neq 17$ at 10% level of significance.

where H_0 : null hypothesis

H_1 : alternate hypothesis

μ : population mean

Let T be average time (in hours) required for the manufacture of a certain type of electronic control panel.

Since n is large, by CLT $\bar{T} \sim N\left(17, \frac{2.0261}{50}\right)$ approximately.

Test Statistic: $Z = \dfrac{\bar{T}-\mu}{\frac{s}{\sqrt{n}}} \sim N(0,1)$

$$= \frac{16.714 - 17}{\left(\sqrt{\frac{2.0261}{50}}\right)} = -1.4208$$

Using GC: p-value = 0.15539

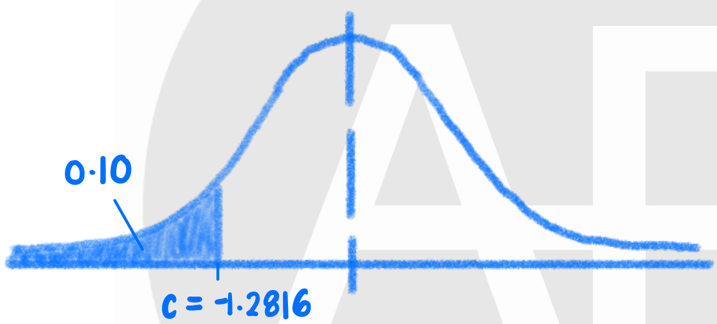
Since p-value > level of significance, we do not reject H_0.
There is insufficient evidence at 10% level of significance to conclude that the average time required has changed.

The Finance Manager wishes to test whether the average time taken for the manufacture of a control panel is shorter using the alternative process. The Finance Manager finds that the average time taken for the manufacture of each of 40 randomly chosen control panels, using the alternative process, is 16.7 hours. He carries out a hypothesis test at the 10% level of significance.

(d) Given that the Finance Manager concludes that there is no reason to reject the null hypothesis, find the range of possible values of the population variance used in calculating the test statistic. [3]

(d) To test $H_0: \mu = 17$ against
$H_1: \mu < 17$

$$z\text{-value} = \frac{16.7-17}{\left(\frac{\sigma}{\sqrt{40}}\right)} = \frac{-0.3\sqrt{40}}{\sigma}$$



Since the financial manager concludes that H_0 is not rejected,

z-value > -1.2816

$$\frac{-0.3\sqrt{40}}{\sigma} > -1.2816$$

$$-0.3\sqrt{40} > -1.2816\,\sigma$$

$$\sigma > \frac{-0.3\sqrt{40}}{-1.2816}$$

$$\sigma > 1.4805$$

$$\therefore \sigma^2 > 2.19$$

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