MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9758/01

Paper 1

October/November 2018

3 hours

5205124461

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

Singapore Examinations and Assessment Board

CAMBRIDGE
International Examinations

JC18 11_9758_01/5R

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1 **(i)** Given that $y = \frac{\ln x}{x}$, find $\frac{dy}{dx}$ in terms of x. [2]

(ii) Hence, or otherwise, find the exact value of $\int_1^e \frac{\ln x}{x^2}\, dx$, showing your working. [4]

(i). $y = \frac{\ln x}{x}$ $\therefore \frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - (\ln x)(1)}{x^2}$

$= \frac{1 - \ln x}{x^2} = \underline{\underline{\frac{1}{x^2} - \frac{\ln x}{x^2}}}$

(ii). $\int_1^e \frac{\ln x}{x^2}\, dx = -\int_1^e \frac{-\ln x}{x^2}\, dx = -\int_1^e \frac{1}{x^2} - \frac{\ln x}{x^2} - \frac{1}{x^2}\, dx$

$= -\int_1^e \left(\frac{1}{x^2} - \frac{\ln x}{x^2}\right) - x^{-2}\, dx$

$= -\left[\frac{\ln x}{x} + \frac{1}{x}\right]_1^e$

$= -\left[\left(\frac{\ln e}{e} + \frac{1}{e}\right) - \left(\frac{\ln 1}{1} + 1\right)\right]$

$= -\left[\frac{2}{e} - 1\right]$

$= \underline{\underline{1 - \frac{2}{e}}}$

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2 **Do not use a calculator in answering this question.**

A curve has equation $y = \frac{3}{x}$ and a line has equation $y + 2x = 7$. The curve and the line intersect at the points A and B.

(i) Find the x-coordinates of A and B. [2]

(ii) Find the exact volume generated when the area bounded by the curve and the line is rotated about the x-axis through 360°. [4]

(i). $y = \frac{3}{x}$ —(1)

$y + 2x = 7$ —(2)

Sub (1) into (2): $\frac{3}{x} + 2x = 7$

$3 + 2x^2 - 7x = 0$

$(2x-1)(x-3) = 0$

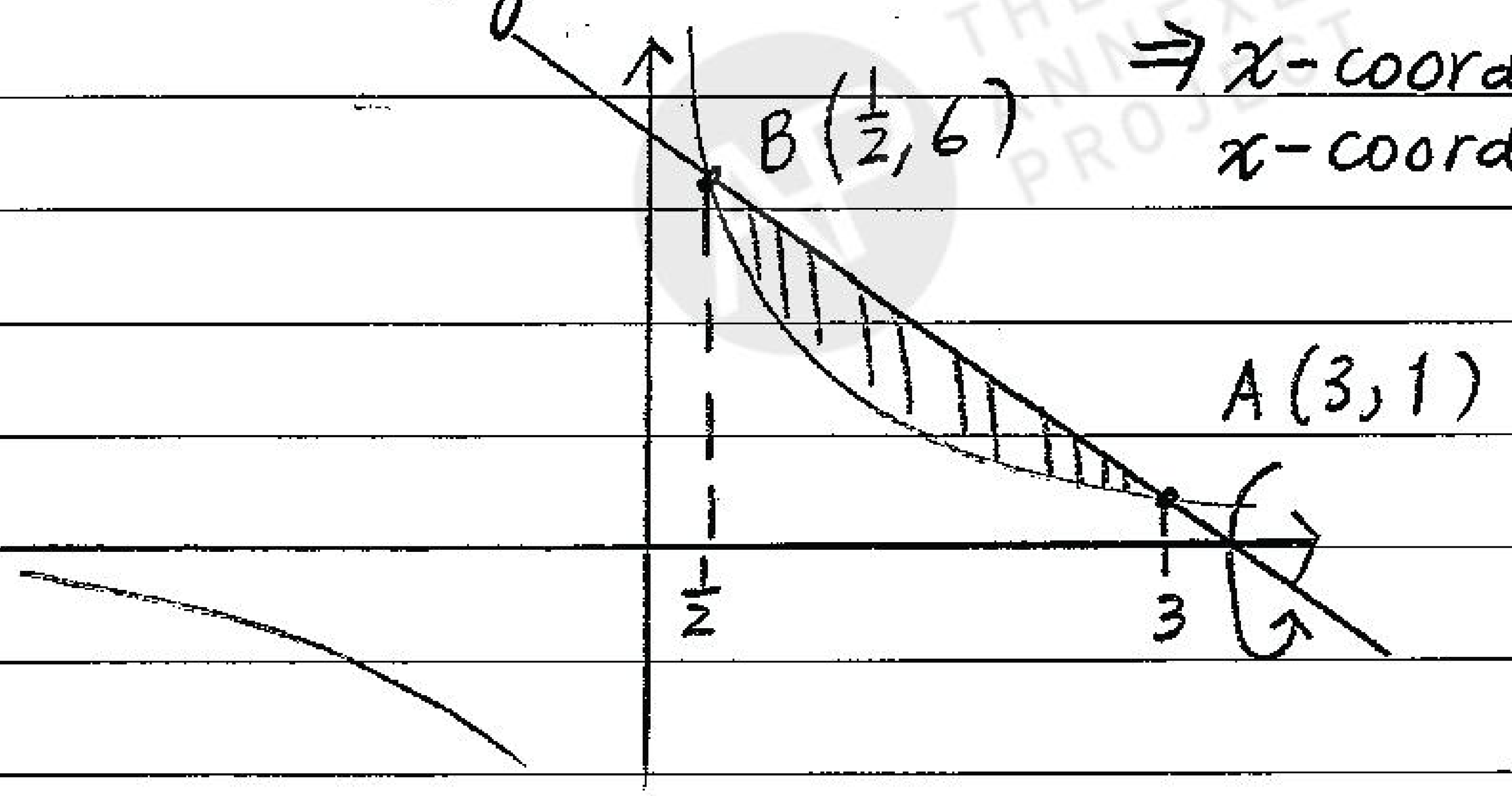
When $x = 3$, $y = 1$ $\therefore A = (3, 1)$

When $x = \frac{1}{2}$, $y = 6$ $\therefore B = (\frac{1}{2}, 6)$

$\Rightarrow$ x-coord. of $A = 3$

x-coord. of $B = \frac{1}{2}$

(ii).



$$\text{Exact Vol} = \pi\int_{1/2}^{3} (-2x+7)^2 - \left(\frac{3}{x}\right)^2 dx$$

$$= \pi\int_{1/2}^{3} 4x^2 - 28x + 49 - 9x^{-2}\, dx$$

$$= \pi\left[\frac{4x^3}{3} - 14x^2 + 49x + \frac{9}{x}\right]_{1/2}^{3}$$

$$= \pi\left[(36 - 126 + 147 + 3) - \left(\frac{1}{6} - \frac{7}{2} + \frac{49}{2} + 18\right)\right]$$

$$= \frac{125\pi}{6} \text{ units}^3.$$

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3 (i) It is given that $x\frac{\mathrm{d}y}{\mathrm{d}x} = 2y - 6$. Using the substitution $y = ux^2$, show that the differential equation can be transformed to $\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{f}(x)$, where the function $\mathrm{f}(x)$ is to be found. [3]

(ii) Hence, given that $y = 2$ when $x = 1$, solve the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x} = 2y - 6$, to find y in terms of x. [4]

(i). given $x\frac{dy}{dx} = 2y - 6$ and $y = ux^2$

differentiate both sides w.r.t x:

$\frac{dy}{dx} = u(2x) + x^2\frac{du}{dx}$

$\therefore x\left[2ux + x^2\frac{du}{dx}\right] = 2ux^2 - 6$

$x^3 \cdot \frac{du}{dx} = -6$

$\underline{\underline{\frac{du}{dx} = \frac{-6}{x^3}}}$

(ii). $\int du = \int -6x^{-3}\, dx$

$u = \frac{-6x^{-2}}{-2} + C$

$u = \frac{3}{x^2} + C$

$\frac{y}{x^2} = \frac{3}{x^2} + C$

$\therefore y = 3 + Cx^2$

Given $y = 2$ when $x = 1$

hence, $2 = 3 + C$

$C = -1$

$\therefore \underline{\underline{y = 3 - x^2}}$

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4 **(i)** Find the exact roots of the equation $|2x^2 + 3x - 2| = 2 - x$. [4]

(ii) On the same axes, sketch the curves with equations $y = |2x^2 + 3x - 2|$ and $y = 2 - x$.

Hence solve exactly the inequality

$$|2x^2 + 3x - 2| < 2 - x.$$ [4]

(i). $|2x^2 + 3x - 2| = 2 - x$

Consider: $2x^2 + 3x - 2 = 2 - x$ and $2x^2 + 3x - 2 = x - 2$

$2x^2 + 4x - 4 = 0$ $\qquad$ $2x^2 + 2x = 0$

$x^2 + 2x - 2 = 0$ $\qquad$ $2x(x+1) = 0$

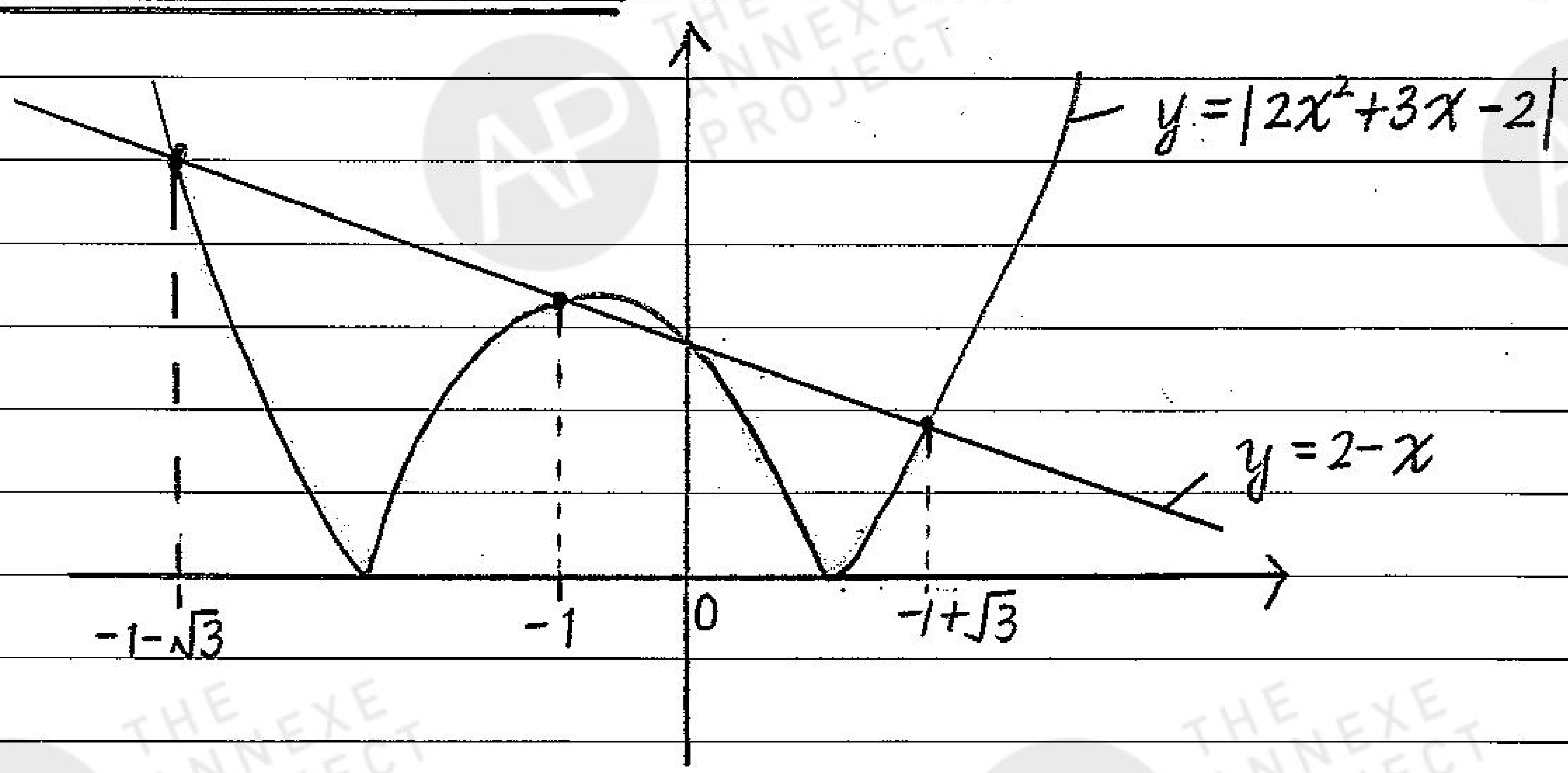
$x = \dfrac{-2 \pm \sqrt{4 - 4(-2)}}{2}$ $\qquad$ $\underline{x = 0 \text{ or } x = -1}$

$= \dfrac{-2 \pm 2\sqrt{3}}{2}$

$= -1 \pm \sqrt{3}$

$= \underline{-1 - \sqrt{3} \text{ or } -1 + \sqrt{3}}$

(ii).



If $|2x^2 + 3x - 2| < 2 - x$

then $\underline{-1 - \sqrt{3} < x < -1 \text{ or } 0 < x < -1 + \sqrt{3}}$

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5 Functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{x+a}{x+b} \quad \text{for } x \in \mathbb{R},\ x \neq -b,\ a \neq -1,$$
$$\mathrm{g}: x \mapsto x \quad \text{for } x \in \mathbb{R}.$$

It is given that ff = g.

Find the value of b.

Find $\mathrm{f}^{-1}(x)$ in terms of x and a. [5]

Given $ff = g$

i.e $f\left(\frac{x+a}{x+b}\right) = x$

$$\frac{\frac{x+a}{x+b} + a}{\frac{x+a}{x+b} + b} = x$$

$$\frac{x+a+a(x+b)}{x+a+b(x+b)} = x$$

$\therefore x + a + \cancel{ax} + ab = x^2 + \cancel{ax} + bx^2 + b^2x$

$(1+b)x^2 + (b^2-1)x - a - ab = 0$

By comparison of coefficients:

$1 + b = 0$

$\underline{\underline{b = -1}}$

Now that $f(x) = \frac{x+a}{x-1}$

$y(x-1) = x + a$

$yx - x = a + y$

$x = \frac{a+y}{y-1}$

$= \frac{y+a}{y-1}$

Hence, $f^{-1}(x) = \underline{\underline{\frac{x+a}{x-1},\ x \in \mathbb{R},\ x \neq 1,\ a \neq -1}}$

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6 Vectors **a**, **b** and **c** are such that $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{a} \times 3\mathbf{b} = 2\mathbf{a} \times \mathbf{c}$.

(i) Show that $3\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$, where λ is a constant. [2]

(ii) It is now given that **a** and **c** are unit vectors, that the modulus of **b** is 4 and that the angle between **b** and **c** is 60°. Using a suitable scalar product, find exactly the two possible values of λ. [5]

(i). Given $\underset{\sim}{a} \times 3\underset{\sim}{b} = 2\underset{\sim}{a} \times \underset{\sim}{c}$

$\therefore (\underset{\sim}{a} \times 3\underset{\sim}{b}) - (2\underset{\sim}{a} \times \underset{\sim}{c}) = 0$

$(\underset{\sim}{a} \times 3\underset{\sim}{b}) - (\underset{\sim}{a} \times 2\underset{\sim}{c}) = 0$

$\underset{\sim}{a} \times (3\underset{\sim}{b} - 2\underset{\sim}{c}) = 0$

Since $\underset{\sim}{a} \times (3\underset{\sim}{b} - 2\underset{\sim}{c}) = 0$

$\underset{\sim}{a} \parallel (3\underset{\sim}{b} - 2\underset{\sim}{c})$

i.e. $(3\underset{\sim}{b} - 2\underset{\sim}{c}) = \lambda \underset{\sim}{a}, \quad \lambda \in \mathbb{R}$

(ii). Given $(3\underset{\sim}{b} - 2\underset{\sim}{c}) = \lambda \underset{\sim}{a}$

$|3\underset{\sim}{b} - 2\underset{\sim}{c}| = |\lambda \underset{\sim}{a}|$

$|3\underset{\sim}{b} - 2\underset{\sim}{c}|^2 = \lambda^2 |\underset{\sim}{a}|^2$

$(3\underset{\sim}{b} - 2\underset{\sim}{c}) \cdot (3\underset{\sim}{b} - 2\underset{\sim}{c}) = \lambda^2$

$9|\underset{\sim}{b}|^2 - 6(\underset{\sim}{b} \cdot \underset{\sim}{c}) - 6(\underset{\sim}{b} \cdot \underset{\sim}{c}) + 4|\underset{\sim}{c}|^2 = \lambda^2$

$9(16) - 12|\underset{\sim}{b}||\underset{\sim}{c}| \cos 60° + 4 = \lambda^2$

$144 - 48(\frac{1}{2}) + 4 = \lambda^2$

$\therefore \lambda^2 = 124$

$\lambda^2 = 4 \times 31$

$\lambda = \pm 2\sqrt{31}$

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7 A curve C has equation $\frac{x^2-4y^2}{x^2+xy^2}=\frac{1}{2}$.

(i) Show that $\frac{dy}{dx}=\frac{2x-y^2}{2xy+16y}$. [3]

The points P and Q on C each have x-coordinate 1. The tangents to C at P and Q meet at the point N.

(ii) Find the exact coordinates of N. [6]

(i). Given $\frac{x^2-4y^2}{x^2+xy^2}=\frac{1}{2}$

$\therefore 2x^2-8y^2=x^2+xy^2$

$x^2-8y^2-xy^2=0$ ——— (1)

differentiate both sides w.r.t x:

$2x-16y\frac{dy}{dx}-x(2y\frac{dy}{dx})+y^2(-1)=0$

$\therefore 16y\frac{dy}{dx}+2xy\frac{dy}{dx}=2x-y^2$

$\therefore \frac{dy}{dx}=\frac{2x-y^2}{2xy+16y}$ (shown).

(ii) When $x=1$,

from (1): $1-8y^2-y^2=0$

$9y^2=1$

$y^2=\frac{1}{9}$

$\therefore y=-\frac{1}{3}$ or $\frac{1}{3}$

Let $P=(1,-\frac{1}{3})$ and $Q=(1,\frac{1}{3})$

at P: $\frac{dy}{dx}=\frac{2-(-\frac{1}{3})^2}{2(\frac{-1}{3})+16(\frac{-1}{3})}=\frac{-17}{54}$

Eqn of tangent at P: $y-(-\frac{1}{3})=\frac{-17}{54}(x-1)$

$y=-\frac{17}{54}x-\frac{1}{54}$ ——(2)

at Q: $\frac{dy}{dx}=\frac{2-(\frac{1}{3})^2}{2(\frac{1}{3})+16(\frac{1}{3})}=\frac{17}{54}$

Eqn of tangent at Q: $y-(\frac{1}{3})=\frac{17}{54}(x-1)$

$y=\frac{17}{54}x+\frac{1}{54}$ ——(2)

Let (2) = (3): $\frac{17}{27}x=\frac{-1}{27}$

$x=\frac{-1}{17}$, $y=0$ $\therefore N=(\frac{-1}{17},0)$

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8 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_{n+1} = 2u_n + An$, where A is a constant and $n \geqslant 1$.

(i) Given that $u_1 = 5$ and $u_2 = 15$, find A and u_3. [2]

It is known that the nth term of this sequence is given by

$$u_n = a(2^n) + bn + c,$$

where a, b and c are constants.

(ii) Find a, b and c. [4]

(iii) Find $\sum_{r=1}^{n} u_r$ in terms of n. (You need not simplify your answer.) [4]

(i). Given $U_{n+1} = 2U_n + An$ and $U_1 = 5$ and $U_2 = 15$

$\therefore \quad 15 = 2(5) + A$

$\underline{\underline{A = 5}}$

also $U_3 = 2U_2 + 2A$

$U_3 = 2(15) + 2(5)$

$= 30 + 10$

$= \underline{\underline{40}}$

(ii). Given $U_n = a(2^n) + bn + c$

Consider $n = 1$: $\quad 5 = a(2) + b + c$

$\therefore \quad 2a + b + c = 5 \quad —(1)$

$n = 2$: $\quad 15 = a(4) + 2b + c$

$\therefore \quad 4a + 2b + c = 15 \quad —(2)$

$n = 3$: $\quad 40 = a(8) + 3b + c$

$\therefore \quad 8a + 3b + c = 40 \quad —(3)$

Using GC: $\underline{\underline{a = 7.5, \quad b = -5, \quad c = -5}}$

(iii). $\sum_{r=1}^{n} \left(\frac{15}{2}\right)2^r - 5r - 5 = \frac{15}{2}\sum_{r=1}^{n} 2^r - 5\sum_{r=1}^{n} r - 5\sum_{r=1}^{n} 1$

$= \frac{15}{2}\left[\frac{2(2^n - 1)}{2-1}\right] - 5\left[\frac{n(n+1)}{2}\right] - 5n$

$= 15(2^n - 1) - \frac{5}{2}n(n+1) - 5n$

$= \underline{\underline{15(2^n) - \frac{5}{2}n^2 - \frac{15}{2}n - 15}}$

9 A curve C has parametric equations

$$x = 2\theta - \sin 2\theta, \qquad y = 2\sin^2\theta,$$

for $0 \leqslant \theta \leqslant \pi$.

(i) Show that $\dfrac{dy}{dx} = \cot\theta$. [4]

(ii) The normal to the curve at the point where $\theta = \alpha$ meets the x-axis at the point A. Show that the x-coordinate of A is $k\alpha$, where k is a constant to be found. [4]

(iii) Do not use a calculator in answering this part.

The distance between two points along a curve is the arc-length. Scientists and engineers need to use arc-length in applications such as finding the work done in moving an object along the path described by a curve or the length of cabling used on a suspension bridge.

The arc-length between two points on C, where $\theta = \beta$ and $\theta = \gamma$, is given by the formula

$$\int_{\beta}^{\gamma} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta.$$

Find the total length of C. [5]

(i). Given $x = 2\theta - \sin 2\theta$ $\qquad$ $y = 2\sin^2\theta$

$\frac{dx}{d\theta} = 2 - 2\cos 2\theta$ $\qquad$ $\frac{dy}{d\theta} = 4\sin\theta\cos\theta$

$= 2 - 2[1 - 2\sin^2\theta]$

$= 2 - 2 + 4\sin^2\theta$

$= 4\sin^2\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{\cancel{4}\cancel{\sin\theta}\cos\theta}{\cancel{4}\sin^{\cancel{2}}\theta} = \cot\theta$ (shown)

(ii). When $\theta = \alpha$, $x = 2\alpha - \sin 2\alpha$, $y = 2\sin^2\alpha$

$\frac{dy}{dx} = \cot\alpha$

Eqn of normal at $\theta = \alpha$:

$y - (2\sin^2\alpha) = \frac{-1}{\cot\alpha}[x - (2\alpha - \sin 2\alpha)]$

at A: $y = 0$

$\therefore -2\sin^2\alpha = -\tan\alpha[x - 2\alpha + \sin 2\alpha]$

$-2\sin^2\alpha = -x\tan\alpha + 2\alpha\tan\alpha - \tan\alpha\sin 2\alpha$

$-\cancel{2\sin^2\alpha} = -x\tan\alpha + 2\alpha\tan\alpha - \frac{\sin\alpha}{\cancel{\cos\alpha}}(\cancel{2\sin\alpha}\cancel{\cos\alpha})$

$\therefore x\tan\alpha = 2\alpha\tan\alpha$

$\underline{x = 2\alpha}$ where $k = 2$

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(iii). $\int_{\beta}^{\gamma} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$

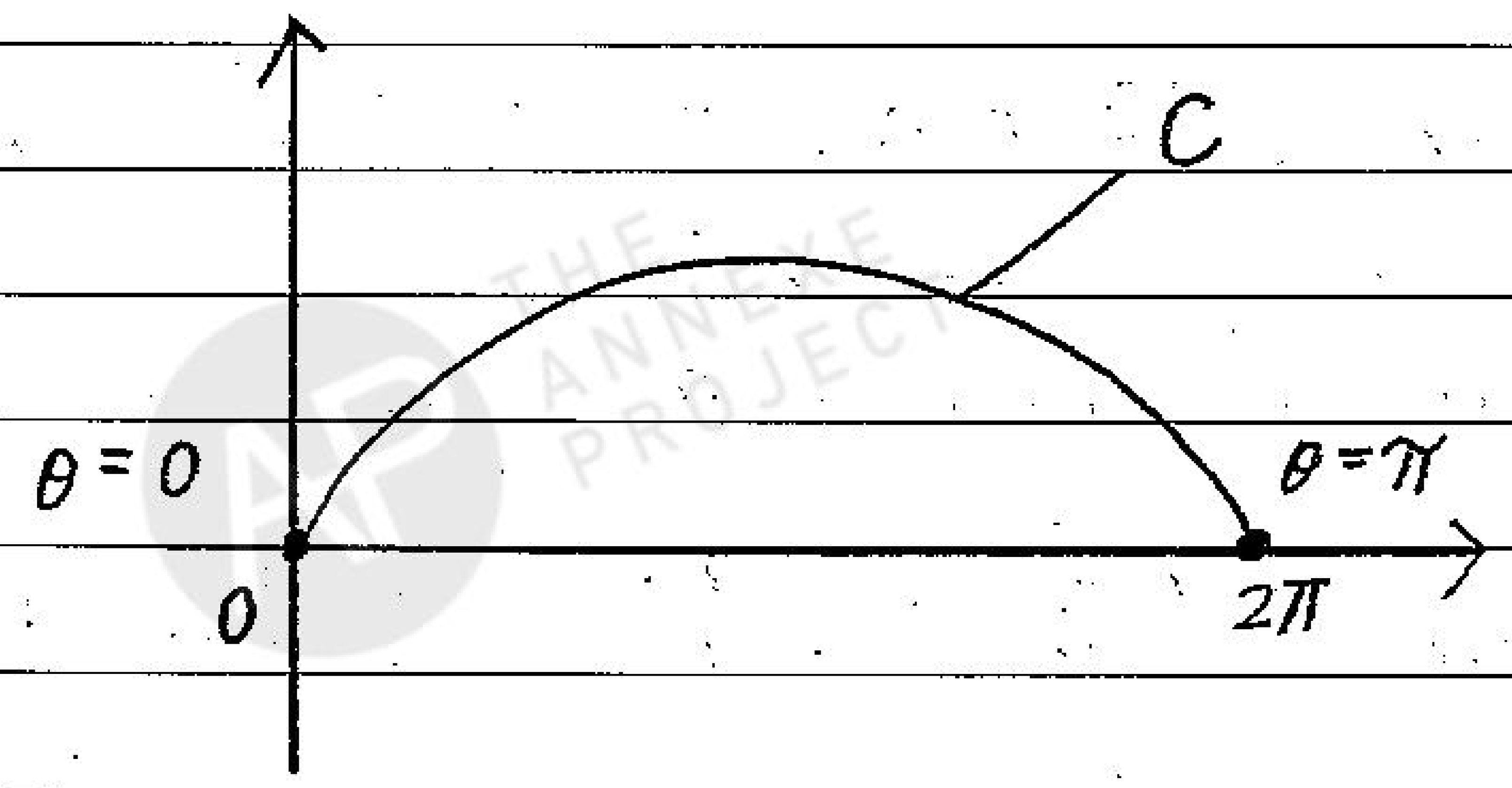
$= \int_{\beta}^{\gamma} \sqrt{(4\sin^2\theta)^2 + (4\sin\theta\cos\theta)^2}\, d\theta$

$= \int_{\beta}^{\gamma} \sqrt{16\sin^4\theta + 16\sin^2\theta\cos^2\theta}\, d\theta$

$= \int_{\beta}^{\gamma} \sqrt{16\sin^2\theta\,(\sin^2\theta + \cos^2\theta)}\, d\theta$

$= \int_{\beta}^{\gamma} 4\sin\theta\, d\theta$

$= \left[-4\cos\theta\right]_{\beta}^{\gamma} = \underline{\underline{-4\cos\gamma + 4\cos\beta}}$



Hence total length of C $= -4\cos\pi + 4\cos 0$

$= 4 + 4 = \underline{\underline{8 \text{ units}}}$

10 An electrical circuit comprises a power source of V volts in series with a resistance of R ohms, a capacitance of C farads and an inductance of L henries. The current in the circuit, t seconds after turning on the power, is I amps and the charge on the capacitor is q coulombs. The circuit can be used by scientists to investigate resonance, to model heavily damped motion and to tune into radio stations on a stereo tuner. It is given that R, C and L are constants, and that $I = 0$ when $t = 0$.

A differential equation for the circuit is $L\frac{dI}{dt} + RI + \frac{q}{C} = V$, where $I = \frac{dq}{dt}$.

(i) Show that, under certain conditions on V which should be stated,

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = 0.$$ [2]

It is now given that the differential equation in part **(i)** holds for the rest of the question.

(ii) Given that $I = Ate^{-\frac{Rt}{2L}}$ is a solution of the differential equation, where A is a positive constant, show that $C = \frac{4L}{R^2}$. [5]

(iii) In a particular circuit, $R = 4$, $L = 3$ and $C = 0.75$. Find the maximum value of I in terms of A, showing that this value is a maximum. [4]

(iv) Sketch the graph of I against t. [2]

(i). Given $L\frac{dI}{dt} + RI + \frac{q}{C} = V$ where $I = \frac{dq}{dt}$

if V is a constant,

differentiate both sides w.r.t t:

then $L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}\frac{dq}{dt} = 0$

$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = 0$ (shown) ——— (1)

(ii).

Given $I = Ate^{-\frac{Rt}{2L}}$

then $\frac{dI}{dt} = (At)e^{-\frac{Rt}{2L}}\left(\frac{-R}{2L}\right) + e^{-\frac{Rt}{2L}}(A)$

$= \left(Ae^{-\frac{Rt}{2L}}\right)\left(-\frac{Rt}{2L} + 1\right)$ ——— (2)

then $\frac{d^2I}{dt^2} = \left(Ae^{-\frac{Rt}{2L}}\right)\left(-\frac{R}{2L}\right) + \left(-\frac{Rt}{2L}+1\right)\left(Ae^{-\frac{Rt}{2L}}\right)\left(-\frac{R}{2L}\right)$

$= \left(Ae^{-\frac{Rt}{2L}}\right)\left(-\frac{R}{2L}\right)\left(2 - \frac{Rt}{2L}\right)$ ——— (3)

Sub (2) & (3) into (1):

$L\left(Ae^{-\frac{Rt}{2L}}\right)\left(-\frac{R}{2L}\right)\left(2-\frac{Rt}{2L}\right) + R\left(Ae^{-\frac{Rt}{2L}}\right)\left(1-\frac{Rt}{2L}\right) + \frac{Ate^{-\frac{Rt}{2L}}}{C} = 0$

divide both sides by $Ae^{-\frac{Rt}{2L}}$:

$\cancel{L}\left(\frac{-R}{2\cancel{L}}\right)\left(2-\frac{Rt}{2L}\right) + R\left(1-\frac{Rt}{2L}\right) + \frac{t}{C} = 0$

$-\cancel{R} + \frac{R^2t}{4L} + \cancel{R} - \frac{R^2t}{2L} = \frac{-t}{C}$

$-\frac{R^2\cancel{t}}{4L} = -\frac{\cancel{t}}{C}$

$\therefore C = \frac{4L}{R^2}$ (shown).

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(iii). Given $R=4$, $L=3$ and $C=0.75$

Since $I = Ate^{-\frac{Rt}{2L}}$

$\therefore I = Ate^{-\frac{4t}{6}}$

$= Ate^{-\frac{2}{3}t}$

$\frac{dI}{dt} = (At)e^{-\frac{2}{3}t}\left(-\frac{2}{3}\right) + \left(e^{-\frac{2}{3}t}\right)(A)$

$= \left(Ae^{-\frac{2}{3}t}\right)\left[-\frac{2}{3}t + 1\right]$

let $\frac{dI}{dt} = 0$

$\therefore \left(Ae^{-\frac{2}{3}t}\right)\left(1 - \frac{2}{3}t\right) = 0$

since A is a positive constant and $e^{-\frac{2}{3}t} > 0$

$\therefore \left(1 - \frac{2}{3}t\right) = 0$

$\frac{2}{3}t = 1$

$t = 1.5$ s

When $t = 1.5$s

$I = A(1.5)e^{-\frac{2}{3}\left(\frac{3}{2}\right)} = 1.5A\left(e^{-1}\right) = \underline{\underline{\frac{1.5A}{e}}}$

Sub above values into (1):

$3\frac{d^2I}{dt^2} + 4(0) + \frac{\frac{1.5A}{e}}{0.75} = 0$

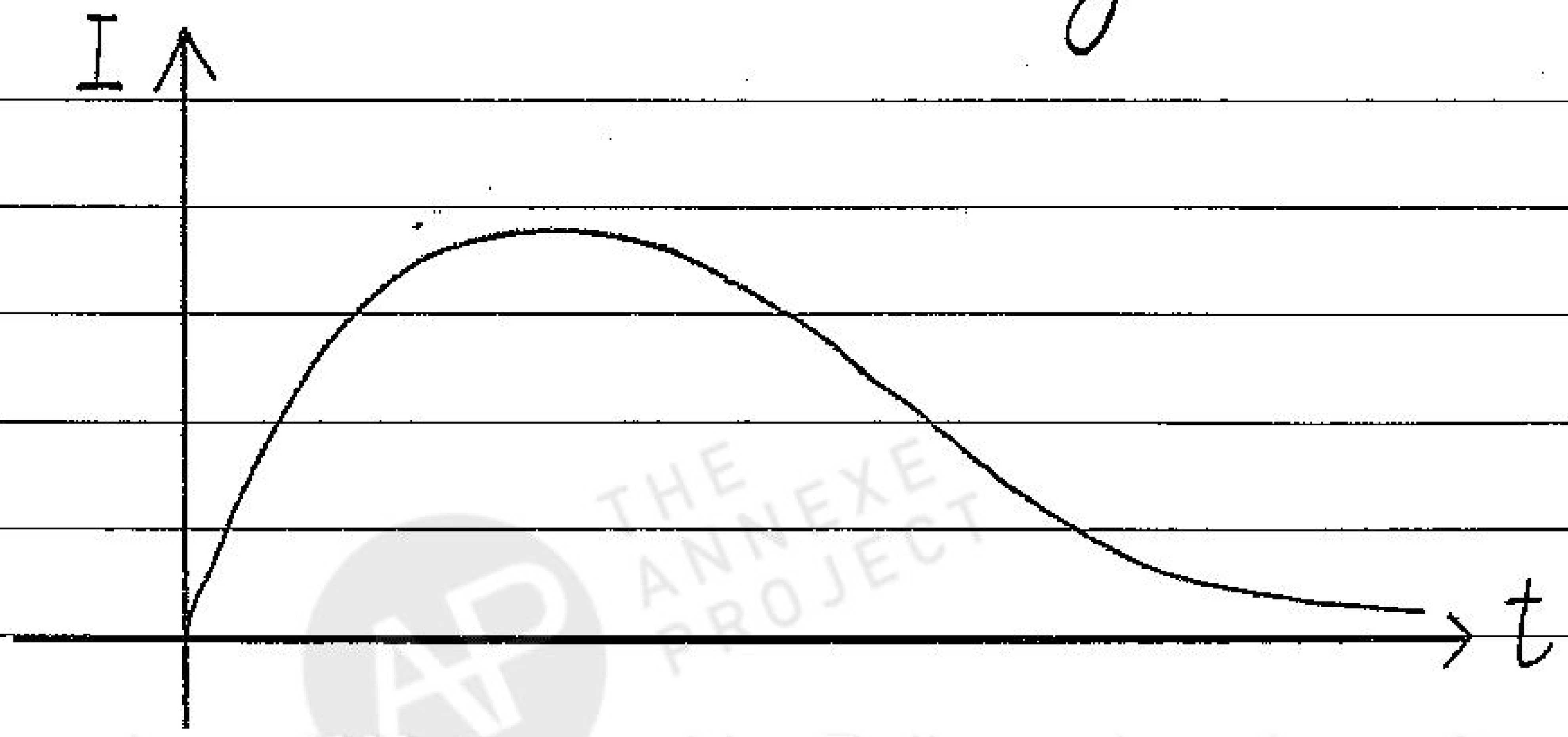
$3\frac{d^2I}{dt^2} = -\frac{\cancel{3}A}{\cancel{2}e} \times \frac{\cancel{4}^{2}}{\cancel{3}}$

$3\frac{d^2I}{dt^2} = -\frac{2A}{e}$

$\therefore \underline{\frac{d^2I}{dt^2} = -\frac{2A}{3e} < 0}$ since A is a positive constant.

Hence, Maximum $I = \frac{1.5A}{e}$, as $\frac{d^2I}{dt^2} < 0$ indicates maximum value by 2nd derivative test.

(iv).



11 Mr Wong is considering investing money in a savings plan. One plan, P, allows him to invest \$100 into the account on the first day of every month. At the end of each month the total in the account is increased by $a\%$.

(i) It is given that $a = 0.2$.

(a) Mr Wong invests \$100 on 1 January 2016. Write down how much this \$100 is worth at the end of 31 December 2016. [1]

(b) Mr Wong invests \$100 on the first day of each of the 12 months of 2016. Find the total amount in the account at the end of 31 December 2016. [3]

(c) Mr Wong continues to invest \$100 on the first day of each month. Find the month in which the total in the account will first exceed \$3000. Explain whether this occurs on the first or last day of the month. [5]

An alternative plan, Q, also allows him to invest \$100 on the first day of every month. Each \$100 invested earns a fixed bonus of \$$b$ at the end of every month for which it has been in the account. This bonus is added to the account. The accumulated bonuses themselves do not earn any further bonus.

(ii) (a) Find, in terms of b, how much \$100 invested on 1 January 2016 will be worth at the end of 31 December 2016. [1]

(b) Mr Wong invests \$100 on the first day of each of the 24 months in 2016 and 2017. Find the value of b such that the total value of all the investments, including bonuses, is worth \$2800 at the end of 31 December 2017. [3]

It is given instead that $a = 1$ for plan P.

(iii) Find the value of b for plan Q such that both plans give the same total value in the account at the end of the 60th month. [3]

(i). (a). Using $A = 100\left(1 + \frac{0.2}{100}\right)^{12}$

$= \$102.43$

(b).

n	amt at the end of nth month
1	$100(1.002)$
2	$[100 + 100(1.002)] \times (1.002)$
	$= 100(1.002) + 100(1.002)^2$
$\vdots$	
n	$100(1.002) + 100(1.002)^2 + \cdots + 100(1.002)^n$
	$= 100\left[\frac{(1.002)(1.002^n - 1)}{1.002 - 1}\right]$

$= 50100(1.002^n - 1)$

When $n = 12$, $A = 50100(1.002^{12} - 1)$

$= \$1215.71$

(c). Let $A \geqslant 3000$

$50100(1.002^n - 1) \geqslant 3000$

By GC: $n \geqslant 29.107$

NO.: Date:

When $n = 29$, i.e 31st May 2018, $A = 50100(1.002^{29} - 1)$
$= \$2988.65$

However, Mr Wong will deposit \$100 on 1st June 2018, hence his account will be at \$3088.65, which exceeds \$3000. Therefore, this occurs on the <u>first of June 2018</u>

(ii). (a). $\$(100 + 12b)$

(b).

n	amt at the end of nth month
1	$100 + b$
2	$(100 + b) + (100 + 2b)$
3	$(100 + b) + (100 + 2b) + (100 + 3b)$
$\vdots$	
24	$24(100) + \{b + 2b + 3b + \cdots + 24b\}$
	$= 2400 + \left[\frac{24}{2}(b + 24b)\right]$
	$= 2400 + 300b$

Let $2800 = 2400 + 300b$

$\therefore 300b = 400$

$b = \frac{4}{3}$

(iii). $100\left[\frac{(1.01)(1.01^{60} - 1)}{1.01 - 1}\right] = 60(100) + \left[\frac{60}{2}(61b)\right]$

$10100(1.01^{60} - 1) = 6000 + 1830b$

$\therefore b = 1.2288$

$= 1.23$