MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER	S					INDEX NUMBER				

ADDITIONAL MATHEMATICS 4049/02

Paper 2

October/November 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

DC (LK/SW) 199932/4

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Show that the equation $3e^x + 5 = 2e^{-x}$ has only one solution and find its value correct to 2 significant figures. [5]

Let $u = e^x$: $3u + 5 = \frac{2}{u}$

$3u^2 + 5u - 2 = 0$

$(3u-1)(u+2) = 0$

$\therefore u = \frac{1}{3}$ or $u = -2$

$e^x = \frac{1}{3}$ or $e^x = -2$

(No Solution as $e^x > 0$ for all real values of x)

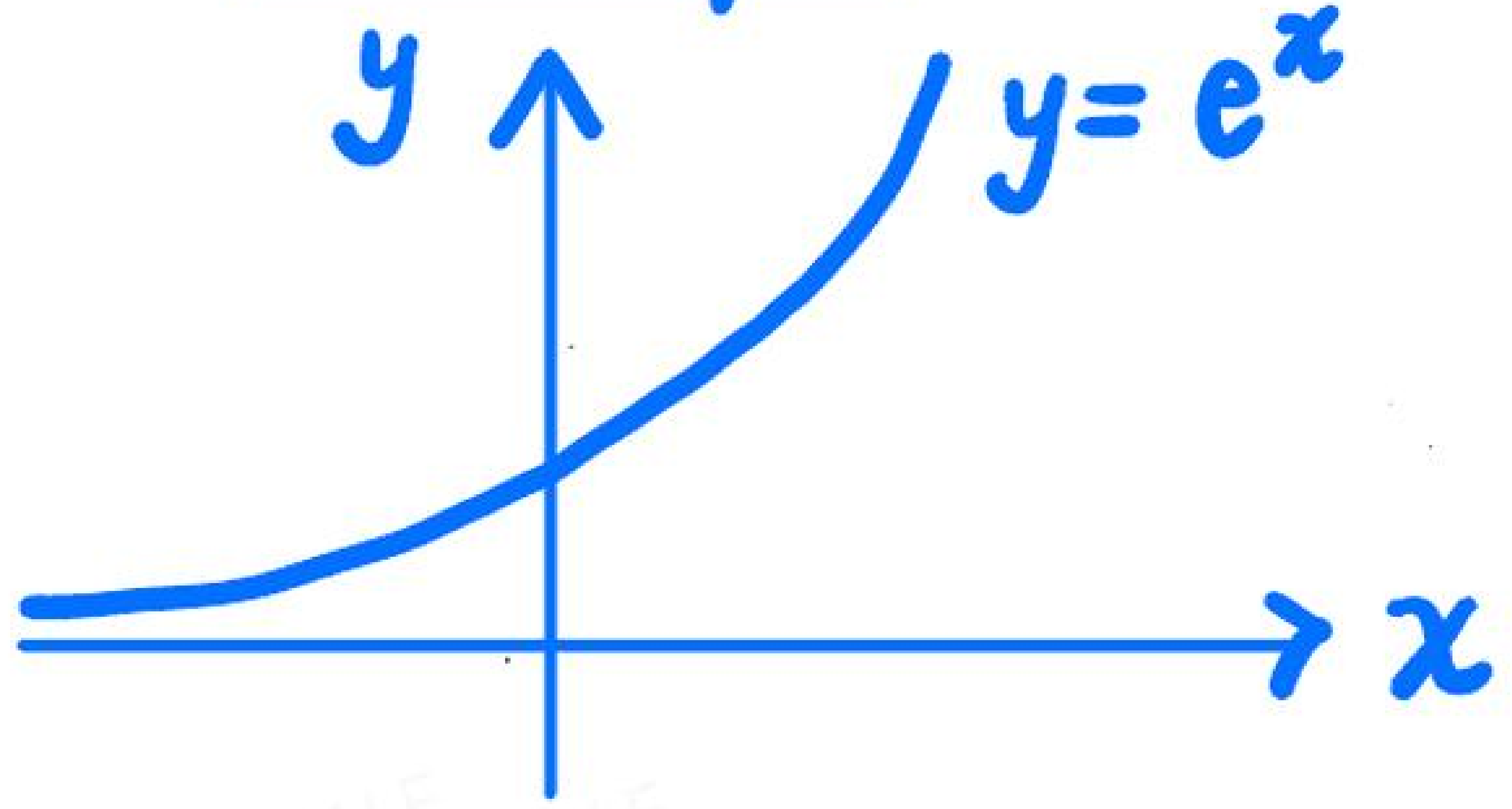
Hence, there is only 1 solution,

i.e. $e^x = \frac{1}{3}$

$x = \ln \frac{1}{3}$

$= -1.1$

See Graph:



2 Show that $x = -1$ is a solution of the equation $3x^3 + 4x^2 - x - 2 = 0$ and hence solve the equation completely. [5]

Let $f(x) = 3x^3 + 4x^2 - x - 2 = 0$

$f(-1) = 3(-1)^3 + 4(-1)^2 - (-1) - 2$

$= -3 + 4 + 1 - 2$

$= 0$

Hence, $x = -1$ is a solution of the equation.

Step 1:

$$
\begin{array}{r|l}
 & 3x^2 + x - 2 \\
\hline
x+1 & 3x^3 + 4x^2 - x - 2 \\
 & -(3x^3 + 3x^2) \\
\hline
 & x^2 - x \\
 & -(x^2 + x) \\
\hline
 & -2x - 2 \\
 & -(-2x - 2) \\
\hline
 & 0
\end{array}
$$

Step 2:

$3x$	-2	$-2x$
		$+$
x	1	$3x$
$3x^2$	-2	x

$\therefore f(x) = (x+1)^2(3x-2)$

For $f(x) = 0$,

$x = -1$ or $\frac{2}{3}$

3 (a) Given that $y = \dfrac{x}{(2x+1)^{\frac{1}{2}}}$, show that $\dfrac{dy}{dx} = \dfrac{x+1}{(2x+1)^{\frac{3}{2}}}$. [4]

$$\frac{dy}{dx} = \frac{(2x+1)^{\frac{1}{2}} \cdot 1 - x \cdot \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2}{(2x+1)}$$

$$= \frac{(2x+1)^{-\frac{1}{2}}\left[(2x+1) - x\right]}{(2x+1)}$$

$$= \frac{x+1}{(2x+1)^{\frac{3}{2}}}$$

Recap Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

(b) Hence find the value of $\displaystyle\int_0^4 \frac{x}{(2x+1)^{\frac{3}{2}}}\,dx$. [4]

$$\int_0^4 \frac{(x+1)-1}{(2x+1)^{\frac{3}{2}}}\,dx$$

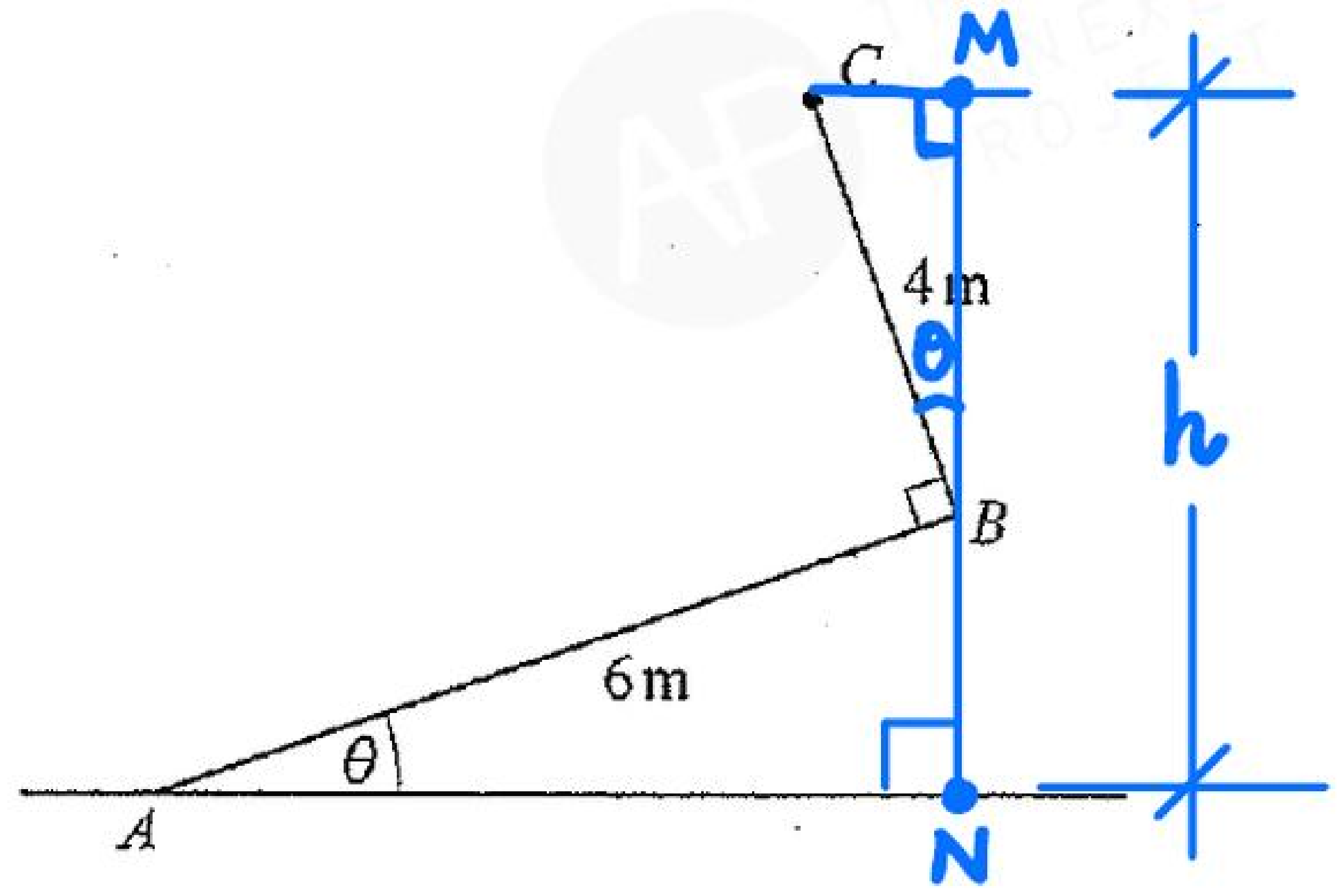
$$= \int_0^4 \frac{x+1}{(2x+1)^{\frac{3}{2}}}\,dx - \int_0^4 \frac{1}{(2x+1)^{\frac{3}{2}}}\,dx$$

$$= \left[\frac{x}{(2x+1)^{\frac{1}{2}}}\right]_0^4 - \int_0^4 (2x+1)^{-\frac{3}{2}}\,dx$$

$$= \left(\frac{4}{3} - 0\right) - \left[\frac{(2x+1)^{-\frac{1}{2}}}{(-\frac{1}{2})(2)}\right]_0^4$$

$$= \frac{4}{3} + \left[(2x+1)^{-\frac{1}{2}}\right]_0^4$$

$$= \frac{4}{3} + \left[\frac{1}{3} - 1\right] = \underline{\frac{2}{3}}$$



The diagram shows two rods AB and BC rigidly joined at B so that angle $ABC = 90°$. The lengths of AB and BC are 6 m and 4 m respectively. A light is positioned at C. The point A is fixed on horizontal ground and the rod AB rotates in a vertical plane with the rod AB inclined at an angle θ to the ground.

(a) Show that the height, h metres, of C above the ground is given by

$$h = 6\sin\theta + 4\cos\theta.$$ [2]

$\angle ABN = 90° - \theta$ (sum of $\angle$s in $\triangle$)

$\angle CBM = 180° - (90° - \theta) - 90° = \theta$ ($\angle$s on a str. line)

$\cos\theta = \frac{MB}{4}$ $\quad\quad$ $\sin\theta = \frac{BN}{6}$

$\therefore MB = 4\cos\theta$ $\quad\quad$ $\therefore BN = 6\sin\theta$

$h = BN + MB = 6\sin\theta + 4\cos\theta$ (shown).

(b) Express h in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° \leqslant \alpha \leqslant 90°$. [4]

$R = \sqrt{6^2 + 4^2}$ $\quad\quad$ $\alpha = \tan^{-1}\frac{4}{6}$

$= \sqrt{52}$ $\quad\quad$ $= 33.690°$

$\therefore h = \sqrt{52}\sin(\theta + 33.690°)$

$= \underline{7.21\sin(\theta + 33.7°)}$

(c) Find the value of h and the corresponding value of θ if the light at C is to be positioned as high as possible. [3]

$h_{max.} = \underline{7.21 \text{ m}}$

when $\theta + 33.7^\circ = 90^\circ$

$\theta = \underline{56.3^\circ}$

5 The equation of a curve is $y = 2x^2 - 6x + 3$.

(a) Find the set of values of x for which the curve lies above the line $y = 11$ and represent this set on a number line. [4]

Step 1:

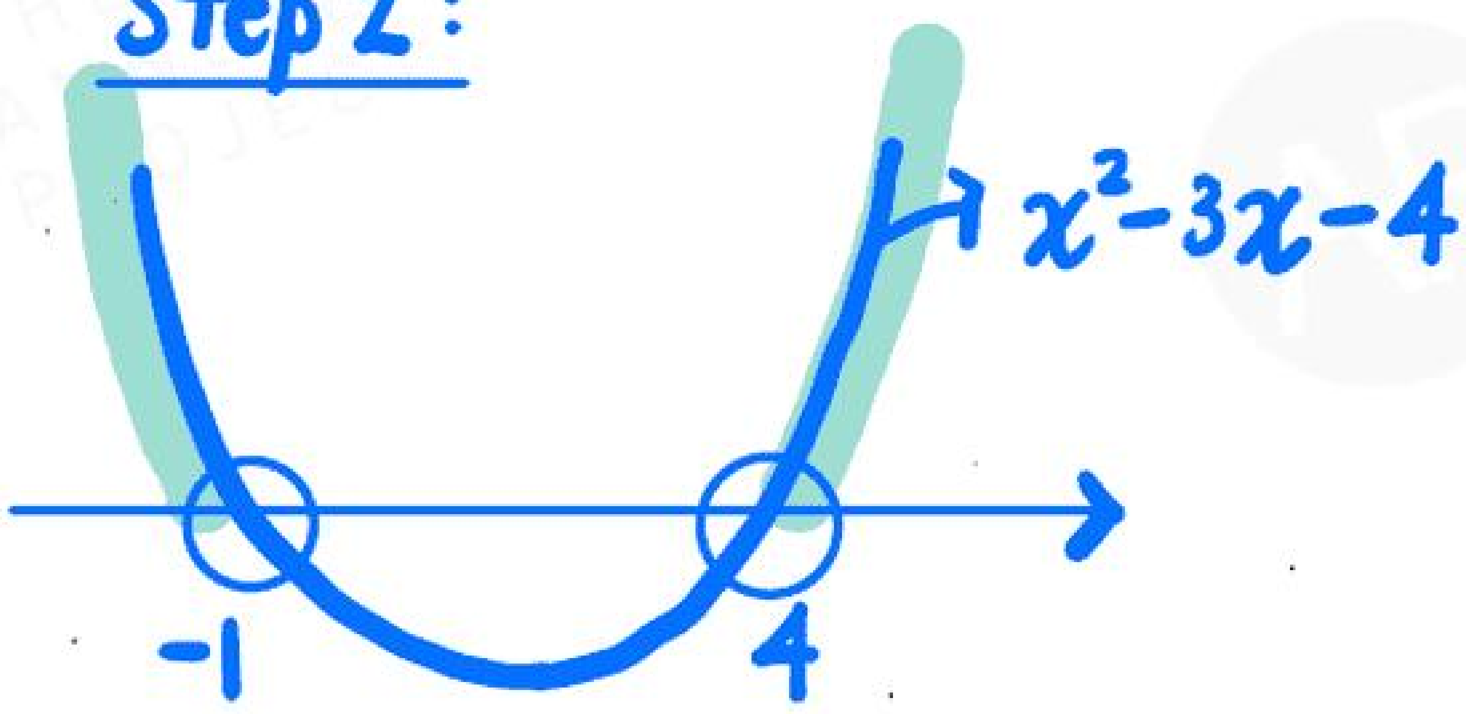
$2x^2 - 6x + 3 > 11$

$2x^2 - 6x - 8 > 0$

$x^2 - 3x - 4 > 0$

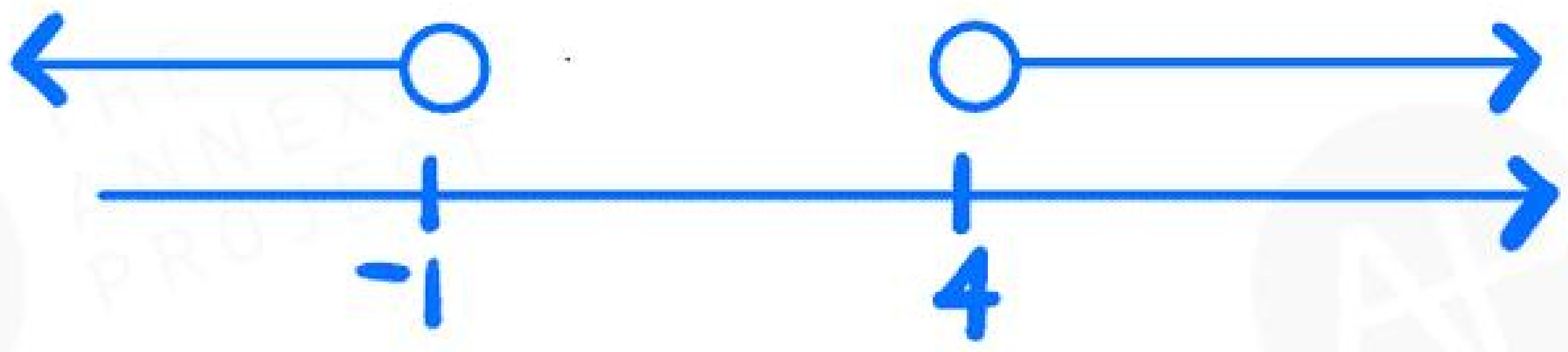
$(x-4)(x+1) > 0$

Step 2:



$x < -1$ or $x > 4$

Step 3:



The line $y = 2x + k$ is a tangent to the curve at the point P.

(b) Find the value of the constant k. [3]

Step 1:

Let $2x^2 - 6x + 3 = 2x + K$

$2x^2 - 8x + (3 - K) = 0$

Step 2:

$b^2 - 4ac = 0$

$(-8)^2 - 4(2)(3-K) = 0$

$64 - 24 + 8K = 0$

$8K = -40$

$\underline{K = -5}$

(c) Find the coordinates of P. [2]

P is the intersection point between the curve and the line.

$\therefore 2x^2 - 6x + 3 = 2x - 5$

$2x^2 - 8x + 8 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$\therefore x = 2$

When $x = 2$, $y = 2(2) - 5 = -1$

$\underline{P = (2, -1)}$

6 (a) Find the terms in $\frac{1}{x^2}$ and $\frac{1}{x^3}$ in the expansion of $\left(2-\frac{3}{x}\right)^6$. [4]

Step 1: We create the general term expression.

$$T_{r+1} = \binom{6}{r} 2^{6-r} \left(\frac{-3}{x}\right)^r$$

$$= \binom{6}{r} 2^{6-r} (-3)^r x^{-r}$$

Step 2:

(a). For term in x^{-2}:

Let $r = 2$: $T_3 = \binom{6}{2} 2^4 (-3)^2 x^{-2}$

$= 2160x^{-2}$

(b). For term in x^{-3}:

Let $r = 3$: $T_4 = \binom{6}{3} 2^3 (-3)^3 x^{-3}$

$= -4320x^{-3}$

Given that there is no term in $\frac{1}{x}$ in the expansion of $(x^2+ax)\left(2-\frac{3}{x}\right)^6$, find the value of the constant a. [2]

$(x^2+ax)[\cdots 2160x^{-2} - 4320x^{-3} + \cdots]$

To obtain a term in $\frac{1}{x}$,
we multiply x^2 with $-4320x^{-3}$,
that results in $-4320x^{-1}$;

we multiply ax with $2160x^{-2}$,
that results in $2160ax^{-1}$.

Given $-4320 + 2160a = 0$

$\underline{a = 2}$

(c) Using the value of a found in part (b), find the coefficient of x in the expansion of $(x^2+ax)\left(2-\frac{3}{x}\right)^6$. [3]

$(x^2+2x)\left[2^6 + 6(2)^5\left(\frac{-3}{x}\right) + 2160x^{-2} - 4320x^{-3} + \cdots\right]$

$\underline{\text{coefficient of } x} = 6(2^5)(-3) + 2(2^6)$

$= \underline{-448}$

7 (a) A formula for working out the stopping distance, d, for a vehicle travelling at a speed v, is $d = av^2 + bv$, where a and b are constants. Values of d for different values of v have been collected. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [4]

$d = av^2 + bv$

$\frac{d}{v} = av + b$

- Plot a graph of $\frac{d}{v}$ (vertical axis) against v (horizontal axis).
- Obtain the gradient value of the straight line plotted, and that is the value of a.
- the y-intercept of the graph is b.

(b) Since 1960, the tiger population in an Asian country has been steadily decreasing. The table shows the estimated number of tigers, n, in the decades following 1960.
The decade 1960–1969 is taken as $t = 1$, and so on.

Year	1960–1969	1970–1979	1980–1989	1990–1999
Value of t	1	2	3	4
Number of tigers n	810	450	240	135

A wild-life expert believed that these figures can be modelled by the formula $n = ab^t$, where a and b are constants.

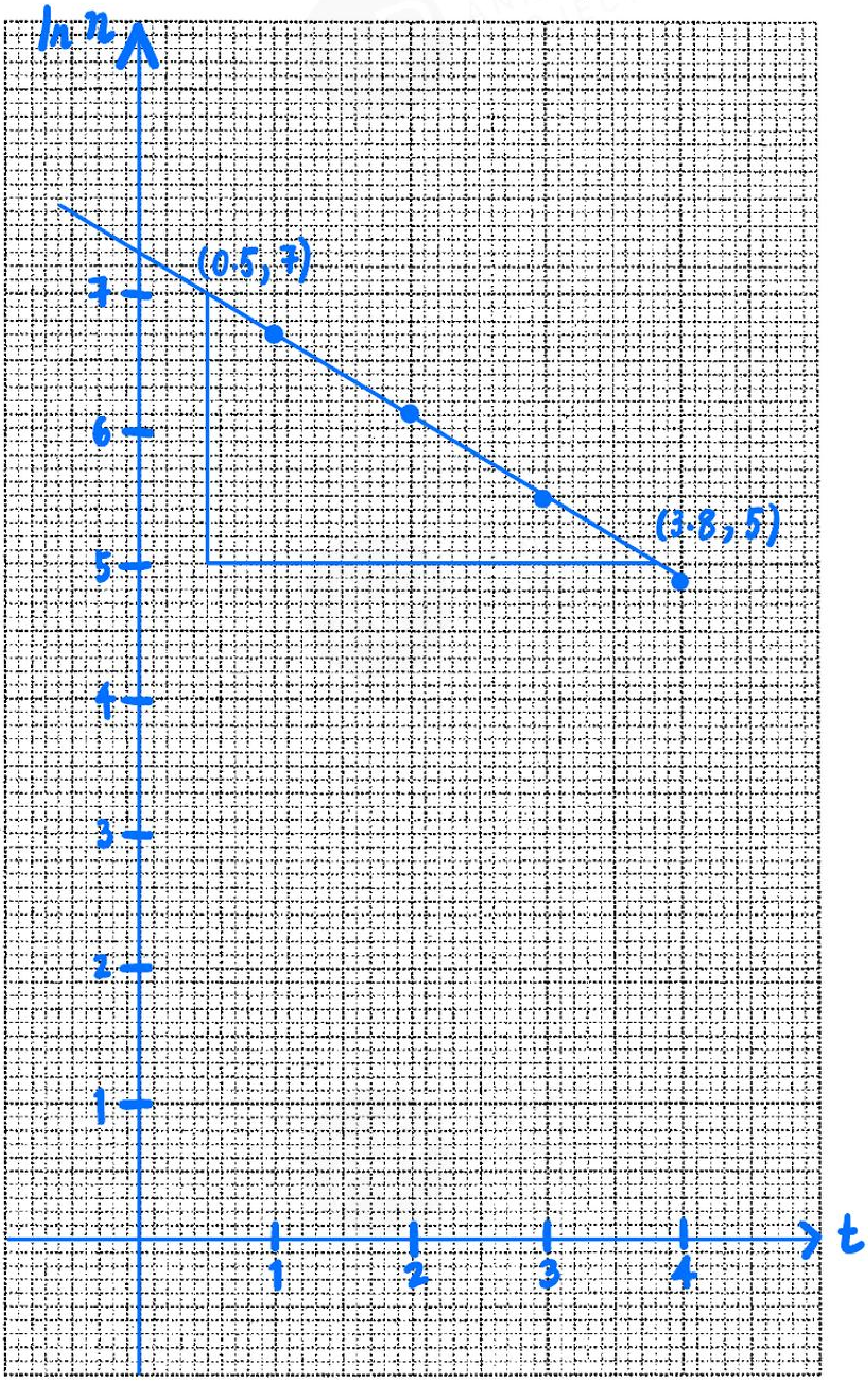
$n = ab^t$

$\ln n = \ln a + t \ln b$

Plot $\ln n$ against t, gradient $= \ln b$, y-intercept $= \ln a$.

$\ln n$	6.70	6.11	5.48	4.91
t	1	2	3	4

(i) Draw a straight line graph to show that the model is reasonable. [4]



from graph,
gradient
$= \frac{7-5}{0.5-3.8}$
$= \frac{-20}{33}$
y-intercept
$= 7.3$

(ii) Estimate the number of tigers in the decade 2000–2009. [2]

$\ln n = \frac{-20}{33} t + 7.3$

When $t = 5$, $\ln n = 4.2697 \Rightarrow n = 71.4999657 \approx \underline{71}$

(iii) Give a reason why this model might not be accurate in later decades. [1]

This model might be inaccurate as the graph was plotted based on data collected from 1960 — 1999.

8 A motorcyclist, travelling at a constant velocity of V m/s, passes a point A and sees roadworks ahead. He immediately applies the brakes and his subsequent velocity, v m/s, is given by $v = 24e^{\frac{-t}{6}}$, where t is the time in seconds after passing A. As he passes a point B his velocity has been halved.

(a) Find the time taken to travel from A to B. [3]

initially, $t=0$: $v = 24e^0 = 24$ m/s

At B: Let $v = 12$ m/s

$$12 = 24e^{-\frac{t}{6}}$$

$$\frac{1}{2} = e^{-\frac{t}{6}}$$

$$\ln\frac{1}{2} = -\frac{1}{6}t$$

$$t = -6\ln\frac{1}{2}$$

$$= 4.1589$$

$$= \underline{4.16 \text{ s}}$$

(b) Find the acceleration of the motorcyclist as he passes B. [3]

$$a = \frac{dv}{dt} = 24e^{-\frac{t}{6}} \cdot \left(-\frac{1}{6}\right) = -4e^{-\frac{t}{6}}$$

When $t = 4.1589$ s,

$$\underline{a = -2.00 \text{ m/s}^2}$$

(c) Find the distance AB. [4]

$$s = \int 24e^{-\frac{t}{6}}\,dt$$

$$= \frac{24e^{-\frac{t}{6}}}{\left(-\frac{1}{6}\right)} + c$$

$$= -144e^{-\frac{t}{6}} + c$$

When $t = 0$ s, $s = 0$ m

$$0 = -144e^{0} + c$$

$$\therefore c = 144$$

$$s = -144e^{-\frac{t}{6}} + 144$$

When $t = 4.1589$ s,

$$s = -144e^{-\frac{4.1589}{6}} + 144$$

$$= \underline{72.0 \text{ m}}$$

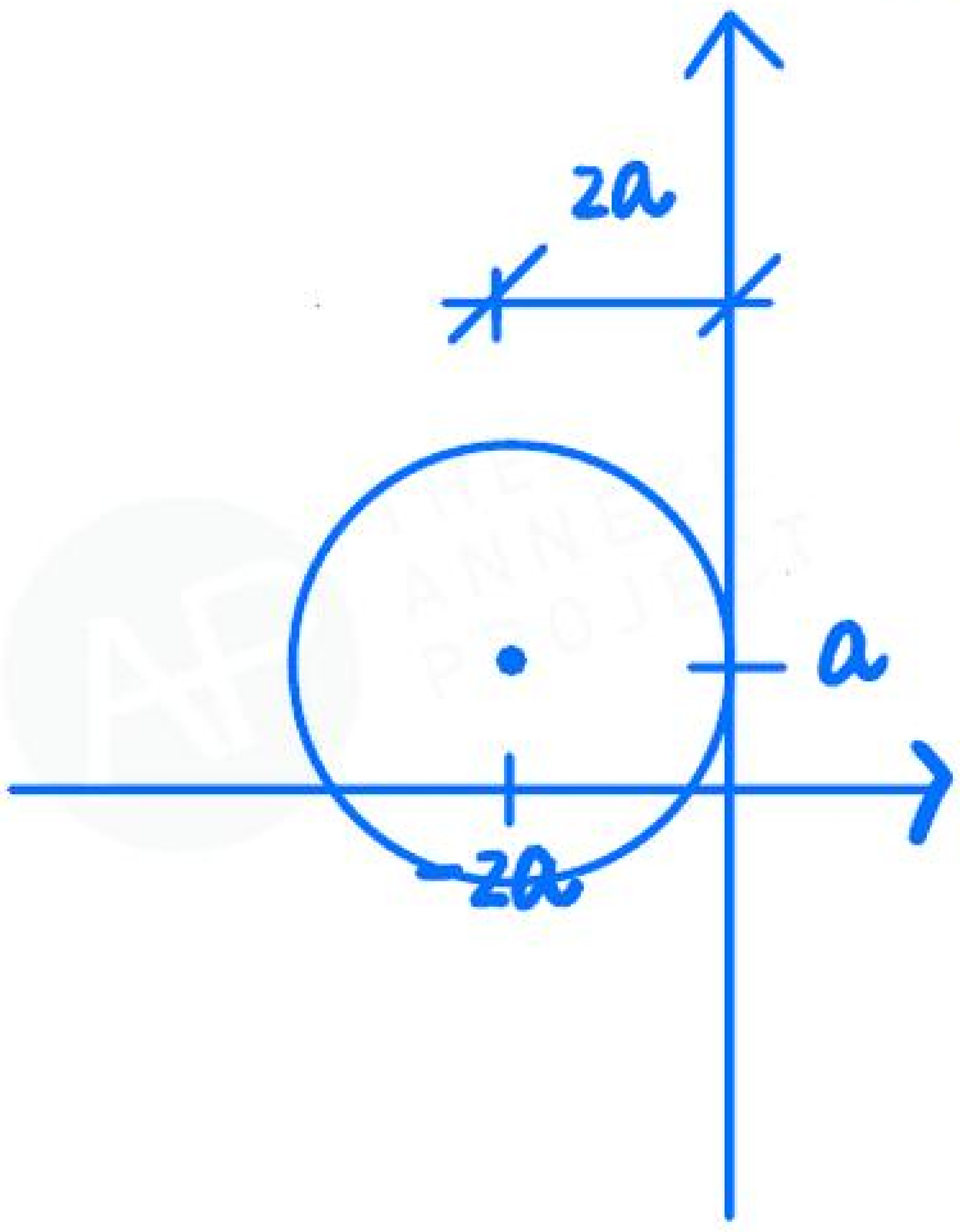
9 The equation of a circle is $(x+2a)^2+(y-a)^2=ka^2$, where a and k are positive constants. It is given that $k=4$.

(a) Explain why the y-axis is a tangent to the circle. [3]

$$[x-(-2a)]^2+(y-a)^2=(2a)^2$$

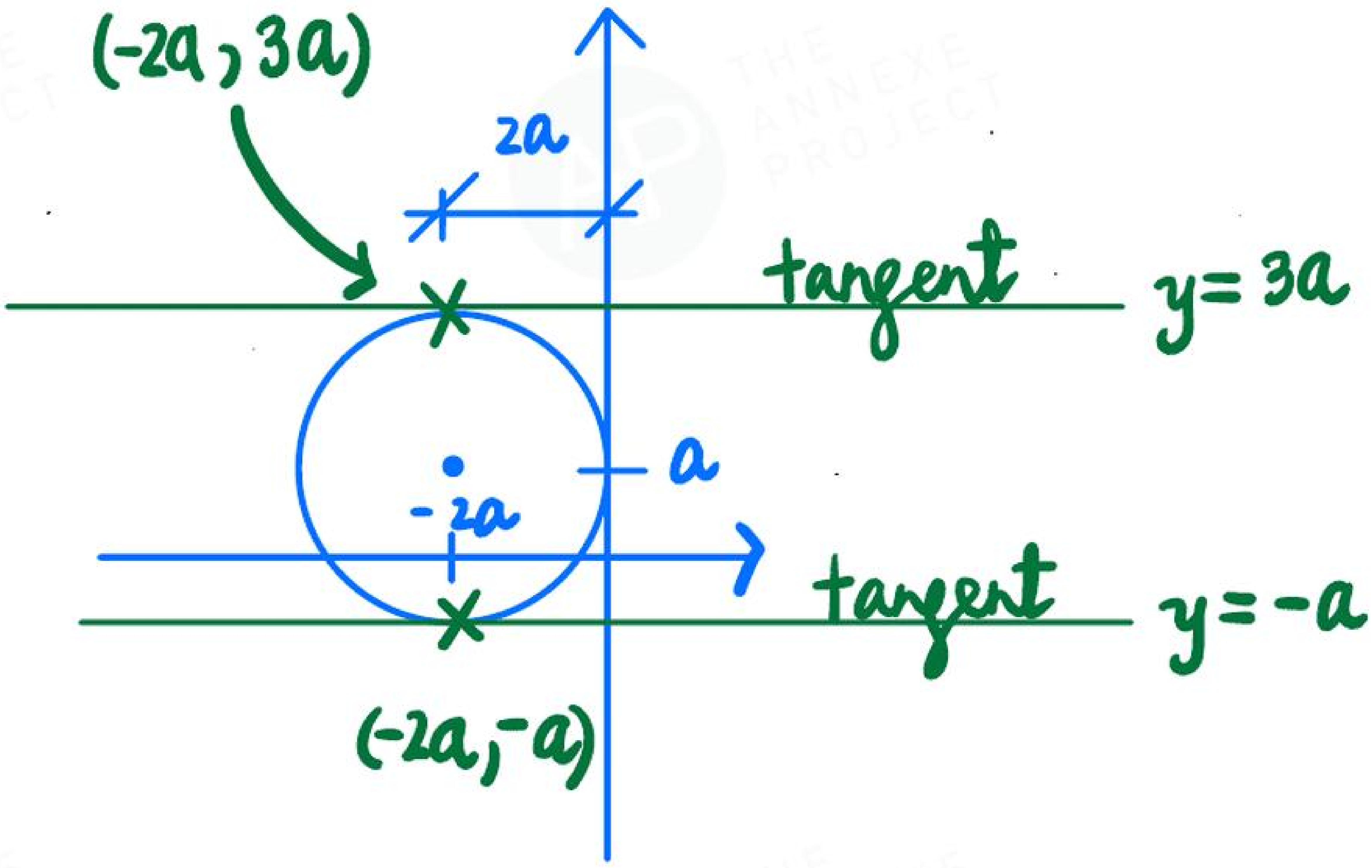
centre of circle $=(-2a, a)$

radius of circle $=2a$



since the radius of the circle is $2a$ units, the horizontal distance of the centre of circle to the y-axis is also $2a$ units, hence, the y-axis is a tangent to the circle as shown in the diagram.

(b) Find, in terms of a, the coordinates of the points on the circle at which the tangent to the circle is parallel to the x-axis. [2]



It is now given that $k = 5$.

(c) Verify that the circle passes through the origin O. [1]

$$(x+2a)^2+(y-a)^2 = 5a^2$$

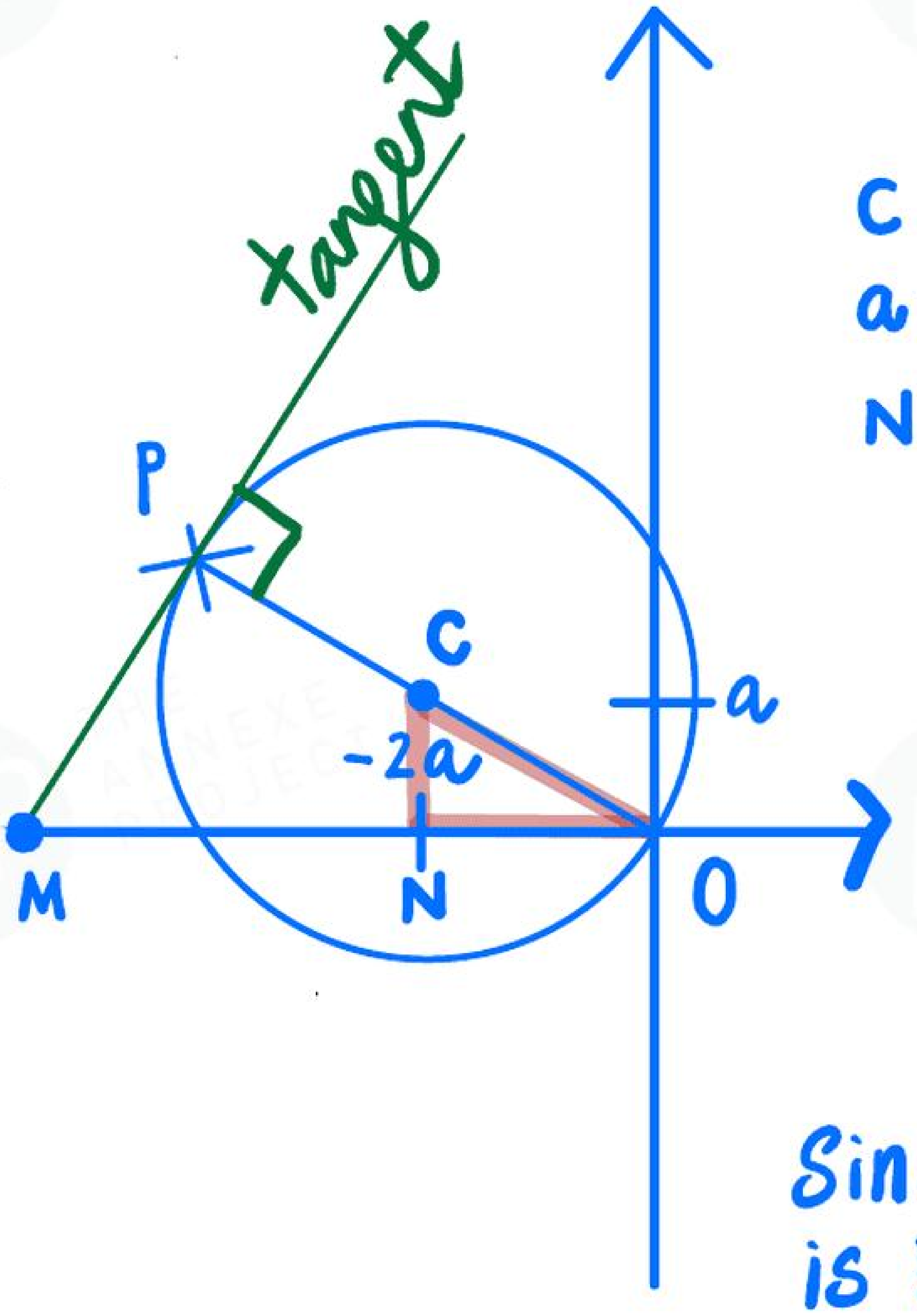
At $(0,0)$: $LHS = (x+2a)^2 + (y-a)^2$

$= (2a)^2 + (-a)^2$

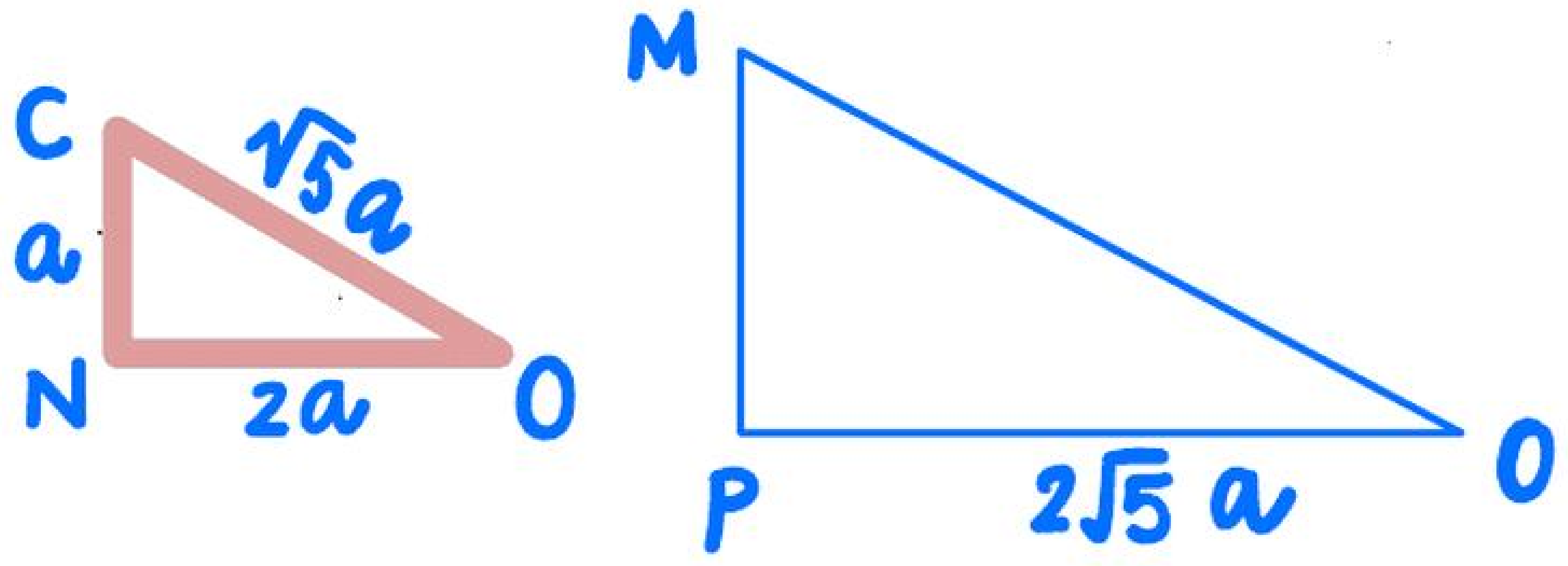
$= 4a^2 + a^2 = 5a^2 = RHS$

Hence, the circle passes through the origin.

(d) Given that OP is a diameter of the circle, find, in terms of a, the coordinates of the point at which the tangent to the circle at P meets the x-axis. [6]



By Similar $\triangle$s :



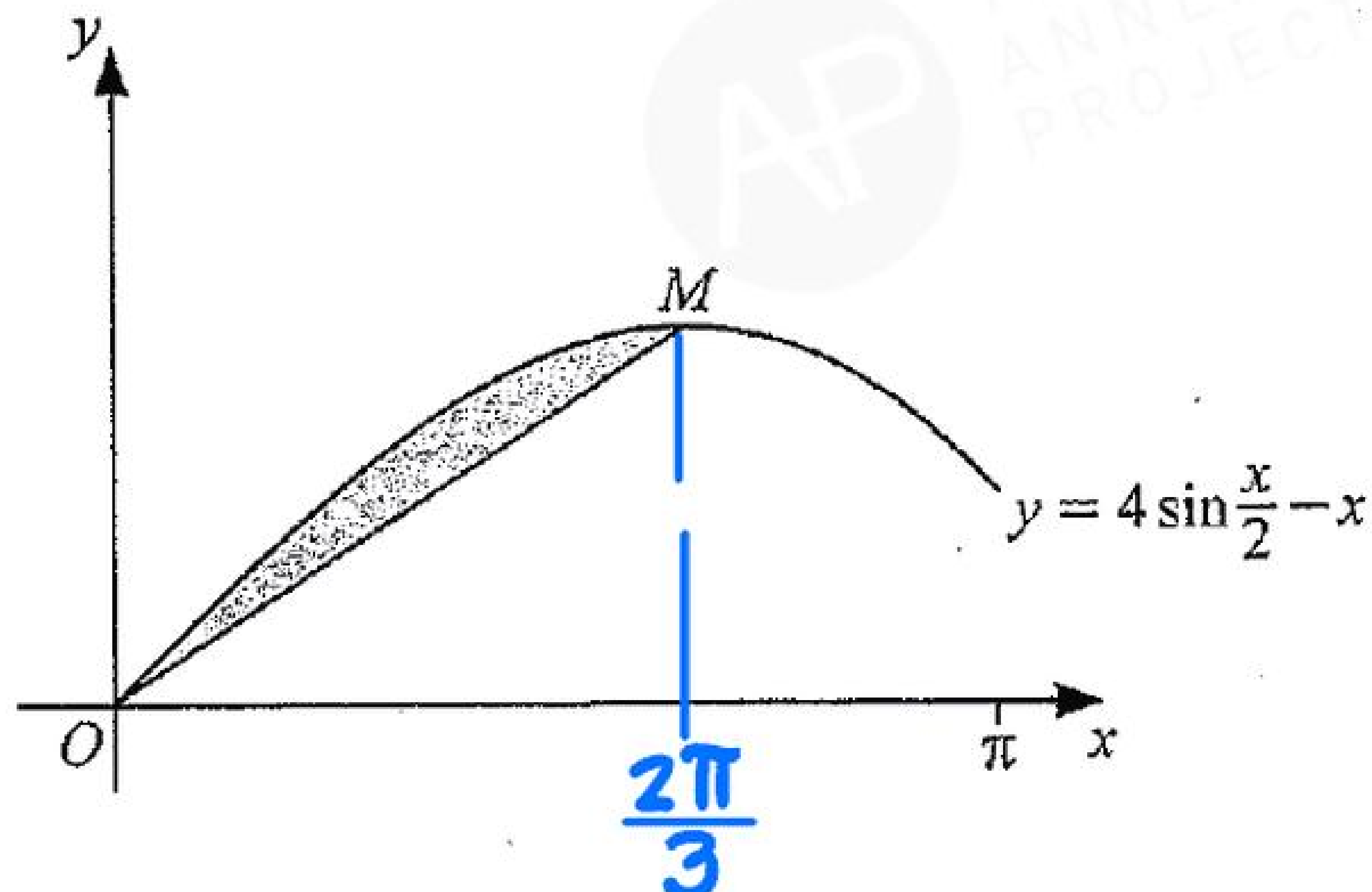
$$\frac{2\cancel{a}}{\sqrt{5}\cancel{a}} = \frac{2\sqrt{5}a}{OM}$$

$$2OM = 10a$$

$$OM = 5a$$

Since the length of OM is $5a$ units,

coordinates of $M = \underline{(-5a, 0)}$



The diagram shows the curve $y = 4\sin\frac{x}{2} - x$ for $0 \leqslant x \leqslant \pi$ radians. The point M is the maximum point of the curve and OM is a straight line.

Show that the area of the shaded region is $4 - \frac{2\pi\sqrt{3}}{3}$ units². [12]

$y = 4\sin\frac{x}{2} - x$

$\frac{dy}{dx} = 4\cos\frac{x}{2} \cdot \frac{1}{2} - 1$

$= 2\cos\frac{x}{2} - 1$

Let $2\cos\frac{x}{2} - 1 = 0$

$\cos\frac{x}{2} = \frac{1}{2}$

basic angle for $\frac{x}{2} = \frac{\pi}{3}$

$\therefore x = \frac{2\pi}{3}$

When $x = \frac{2\pi}{3}$,

$y = 4\sin\frac{\pi}{3} - \frac{2\pi}{3}$

$= \frac{4\sqrt{3}}{2} - \frac{2\pi}{3}$

$= 2\sqrt{3} - \frac{2\pi}{3} \Rightarrow \underline{M = \left(\frac{2\pi}{3}, 2\sqrt{3} - \frac{2\pi}{3}\right)}$

Continuation of working space for question 10.

$$\text{Shaded Area} = \int_0^{\frac{2\pi}{3}} 4\sin\tfrac{1}{2}x - x \, dx - \left[\tfrac{1}{2} \times \tfrac{2\pi}{3} \times \left(2\sqrt{3} - \tfrac{2\pi}{3}\right)\right]$$

$$= \left[\frac{-4\cos\frac{1}{2}x}{\left(\frac{1}{2}\right)} - \frac{x^2}{2}\right]_0^{\frac{2\pi}{3}} - \left[\frac{\pi}{3}\left(2\sqrt{3} - \frac{2\pi}{3}\right)\right]$$

$$= \left[-8\cos\frac{\pi}{3} - \cancel{\frac{2\pi^2}{9}}\right] - [-8] - \left[\frac{2\sqrt{3}\pi}{3} - \cancel{\frac{2\pi^2}{9}}\right]$$

$$= -4 + 8 - \frac{2\sqrt{3}\pi}{3}$$

$$= \underline{4 - \frac{2\sqrt{3}\pi}{3}} \text{ units}$$

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