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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER	S					INDEX NUMBER				

ADDITIONAL MATHEMATICS

4049/02

Paper 2

October/November 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

DC (CE/SW) 301928/6

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 In 2010, there were an estimated 2400 birds of a particular species in a certain country. In 2013, the numbers were estimated to have fallen to 2050. Scientists believe that the number of these birds, N, can be modelled by the formula $N = 2400e^{-kt}$, where t is the time in years after 2010.

(a) Estimate the number of birds in the year 2017. [4]

When $t = 3$, $N = 2050$

$$\therefore 2400e^{-3K} = 2050$$

$$e^{-3K} = \frac{2050}{2400} = \frac{41}{48}$$

$$-3K \ln e = \ln \frac{41}{48}$$

$$-3K = \ln \frac{41}{48}$$

$$K = \underline{0.052543}$$

When $t = 7$, $N = 2400e^{-0.052543(7)}$

$$= 1661.4$$

$$= \underline{1660 \text{ (3 s.f.)}}$$

(b) The species is labelled "under threat" when the number of birds falls below 1200. Estimate the year in which the species might be first labelled "under threat". [2]

Let $1200 = 2400e^{-0.052543t}$

$$0.5 = e^{-0.052543t}$$

$$\ln 0.5 = -0.052543t$$

$$\therefore t = 13.192$$

estimated year $= 2010 + 14$

$= \underline{2024}$

2 It is given that $f(x) = x^3 + ax^2 - 3x + b$, where a and b are constants, has a factor of $x+2$ and leaves a remainder of 30 when divided by $x-3$.

(a) Find the values of a and b. [4]

$f(-2) = -8 + 4a + 6 + b = 0$

$\therefore b = 2 - 4a$ —— ①

$f(3) = 27 + 9a - 9 + b = 30$

$\therefore b = 12 - 9a$ —— ②

Solving ① & ②:

$2 - 4a = 12 - 9a$

$5a = 10$

$a = 2$

$\therefore b = 2 - 4(2) = -6$

(b) Using these values of a and b, factorise $f(x)$ completely into three linear factors, using surds where necessary. [3]

Step 1: Long Division

$$\begin{array}{r} x^2 - 3 \\ x+2 \overline{\smash{)}\, x^3 + 2x^2 - 3x - 6} \\ -(x^3 + 2x^2) \\ \hline -3x - 6 \\ -(-3x - 6) \\ \hline 0 \end{array}$$

Step 2:

$f(x) = (x+2)(x^2-3)$

$= (x+2)(x-\sqrt{3})(x+\sqrt{3})$

3 (a) Prove the identity $\tan 2\theta(2\cos\theta - \sec\theta) = 2\sin\theta$. [4]

$$\text{LHS} = \tan 2\theta(2\cos\theta - \sec\theta)$$
$$= \frac{\sin 2\theta}{\cos 2\theta}\left(2\cos\theta - \frac{1}{\cos\theta}\right)$$
$$= \frac{2\sin\theta\cos\theta}{\cancel{2\cos^2\theta - 1}} \times \frac{\cancel{2\cos^2\theta - 1}}{\cos\theta}$$
$$= \frac{2\sin\theta\cancel{\cos\theta}}{\cancel{\cos\theta}}$$
$$= \underline{2\sin\theta} = \text{RHS (shown)}$$

(b) Hence solve the equation $\tan 2\theta(2\cos\theta - \sec\theta) = \frac{1}{2}\sec\theta$ for $0° \leqslant \theta \leqslant 360°$. [4]

$$\tan 2\theta(2\cos\theta - \sec\theta) = \frac{1}{2}\sec\theta$$

from part (a): $2\sin\theta = \frac{1}{2\cos\theta}$

$$2(2\sin\theta\cos\theta) = 1$$
$$\sin 2\theta = \frac{1}{2}$$

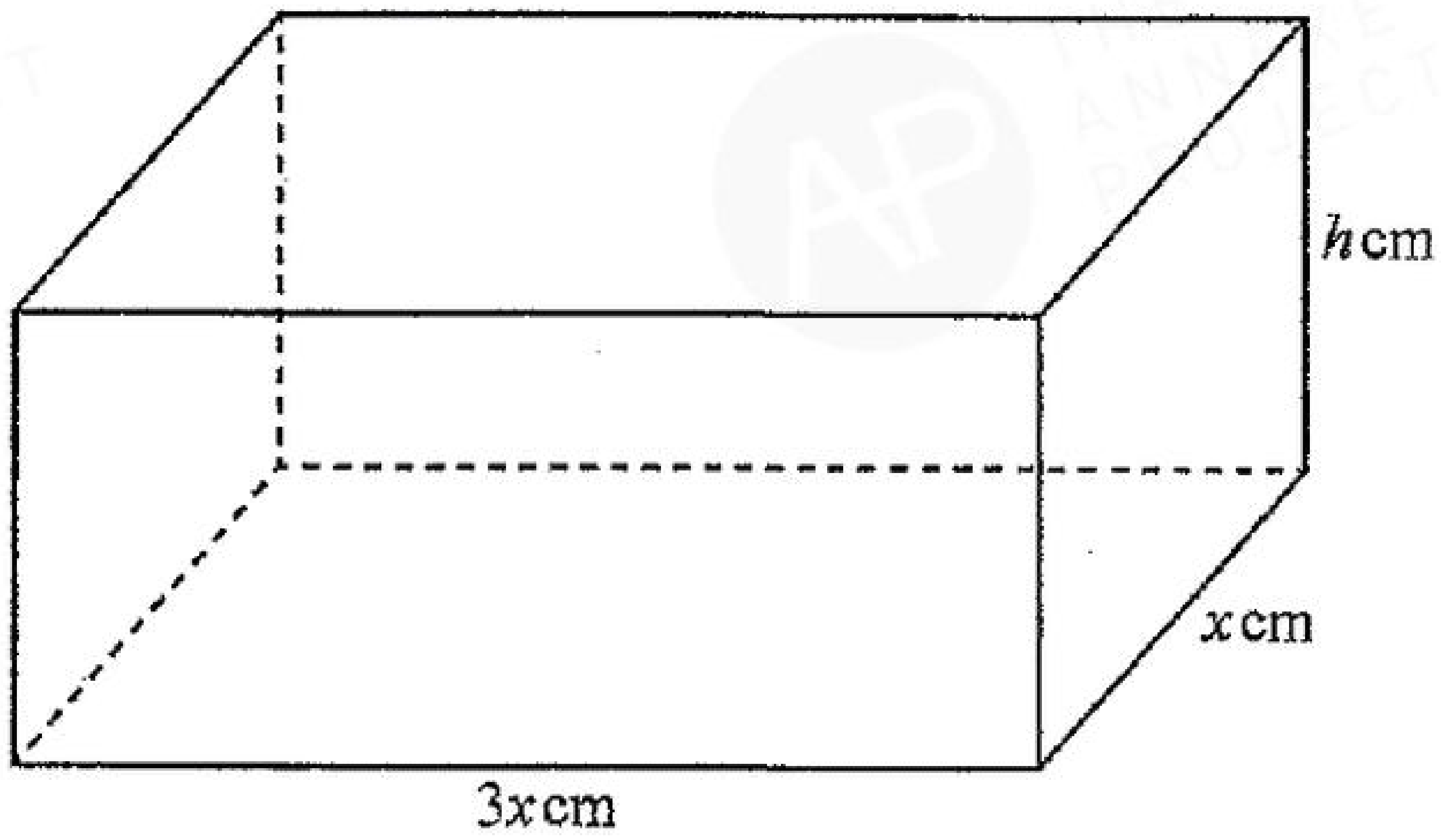
Step 1: Basic $\angle$ for $2\theta = \sin^{-1}\frac{1}{2}$
$= 30°$

Step 2: 2θ lies in 1^{st} or 2^{nd} quad.

Step 3: $0° \leqslant 2\theta \leqslant 720°$

Hence, $2\theta = 30°, 150°, 390°$ or $510°$

$\underline{\theta = 15°, 75°, 195° \text{ or } 255°}$



The diagram shows a closed food container designed in the shape of a cuboid. The rectangular base has dimensions of x cm and $3x$ cm. The height of the container is h cm. The volume of the container is $2000\,\text{cm}^3$.

(a) Show that the total surface area of the six faces of the container is $6x^2 + \dfrac{16000}{3x}$. [4]

$$3x \times x \times h = 2000$$

$$3hx^2 = 2000$$

$$h = \frac{2000}{3x^2} \quad \text{——— } (1)$$

$$A = 2(3xh) + 2(xh) + 2(3x^2)$$

$$= 8hx + 6x^2$$

$$= 8\left(\frac{2000}{3x^2}\right)x + 6x^2$$

$$= \underline{6x^2 + \frac{16000}{3x}} \quad \text{(shown)}$$

(b) Given that x can vary, find the value of x for which the total surface area is stationary. [4]

Step 1: $A = 6x^2 + \frac{16000}{3} x^{-1}$

$$\frac{dA}{dx} = 12x - \frac{16000}{3} x^{-2}$$

$$= 12x - \frac{16000}{3x^2}$$

$$= \frac{36x^3 - 16\,000}{3x^2}$$

Step 2: Let $\frac{dA}{dx} = 0$

$$36x^3 - 16\,000 = 0$$

$$x^3 = \frac{4000}{9}$$

$$\therefore x = 7.6314$$

$$= \underline{7.63}$$

(c) Determine whether this value of x gives a maximum or minimum value for the total surface area of the container. [2]

$$\frac{dA}{dx} = 12x - \frac{16\,000}{3} x^{-2}$$

$$\frac{d^2A}{dx^2} = 12 + \frac{32000}{3} x^{-3}$$

$$= 12 + \frac{32000}{3x^3}$$

When $x = 7.63$, $\frac{d^2A}{dx^2} = 12 + \frac{32000}{3(7.6314)^3}$

$$> 0$$

By 2nd derivative test, $x = 7.63$ gives a <u>minimum value</u> to A.

5 **A calculator must not be used in this question.**

(a) Show that $\tan 75° = 2+\sqrt{3}$. [4]

$$\tan 75° = \tan(45° + 30°)$$

$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

• multiply both numerator and denominator by $\sqrt{3}$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$\boxed{\text{rationalise}}$

$$= \frac{3 + 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$= \underline{2 + \sqrt{3}}$$

(b) Use the result from **part (a)** to find an expression for $\sec^2 75°$, in the form $a + b\sqrt{3}$ where a and b are integers. [2]

$$\sec^2 75° = 1 + \tan^2 75°$$
$$= 1 + (2+\sqrt{3})^2$$
$$= 1 + 4 + 4\sqrt{3} + 3$$
$$= \underline{8 + 4\sqrt{3}}$$

(c) Hence express $\cos^2 75°$ in the form $c + d\sqrt{3}$, where c and d are fractions. [2]

$$\cos^2 75° = \frac{1}{\sec^2 75°}$$
$$= \frac{1}{8+4\sqrt{3}}$$
$$= \frac{1}{8+4\sqrt{3}} \times \frac{8-4\sqrt{3}}{8-4\sqrt{3}}$$
$$= \frac{8-4\sqrt{3}}{8^2-(4\sqrt{3})^2}$$
$$= \frac{8-4\sqrt{3}}{64-48}$$
$$= \underline{\frac{8-4\sqrt{3}}{16} \text{ or } \frac{1}{2} - \frac{\sqrt{3}}{4}}$$

6 (a) Solve the equation $2^x + 4^{x-1} = 15$. [5]

$2^x + (2^2)^{x-1} = 15$

$2^x + 2^{2x-2} = 15$

$2^x + \frac{2^{2x}}{2^2} = 15$

Let $2^x = a$:

$\therefore a + \frac{a^2}{4} = 15$

$4a + a^2 = 60$

$a^2 + 4a - 60 = 0$

$(a+10)(a-6) = 0$

$\therefore a = -10$ or $a = 6$

i.e. $2^x = -10$ or $2^x = 6$

(No Solution) $\lg 2^x = \lg 6$

$x \lg 2 = \lg 6$

$x = \frac{\lg 6}{\lg 2}$

$= \underline{2.58 \text{ (3 s.f.)}}$

(b) The curve $y = 5 - e^{2x}$ intersects the x-axis at A and the y-axis at B.

(i) Find the coordinates of A and B, in logarithmic form where appropriate. [2]

Find A: Let $y = 0$

$5 - e^{2x} = 0$

$e^{2x} = 5$

$\ln e^{2x} = \ln 5$

$2x = \ln 5$

$x = \frac{1}{2}\ln 5$

$\therefore A = (\frac{1}{2}\ln 5, 0)$

Find B: Let $x = 0$

$y = 5 - e^0$

$= 4$

$\therefore B = (0, 4)$

(ii) Find the gradient of the curve at A. [2]

Step 1: $\frac{dy}{dx} = -2e^{2x}$

Step 2: When $x = \frac{1}{2}\ln 5$,

$\frac{dy}{dx} = -2e^{2(\frac{1}{2}\ln 5)}$

$= -2e^{\ln 5}$

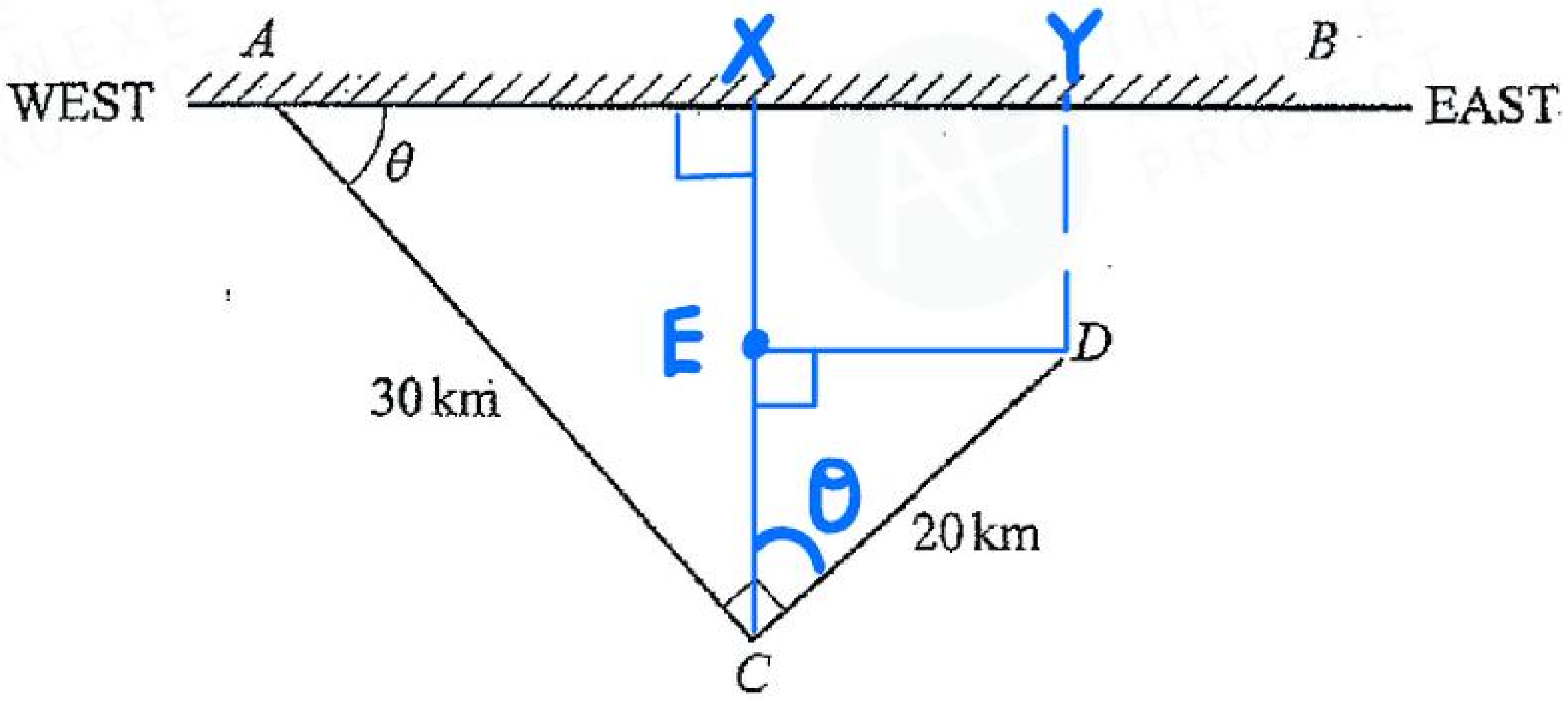
$= -2(5)$

$= -10$

Shortcut Formula:

$e^{\ln a} = a$

7



Two countries are separated by a border that runs from west to east. Villages A and B lie on the bord A helicopter flies 30 km from village A to a village C. Angle BAC is $\theta°$. The helicopter then trav 20 km to a village D at an angle of 90° to AC as shown. You can assume that the helicopter always fl at the same height.

(a) Show that the shortest distance of village D from the border is $(30\sin\theta - 20\cos\theta)$ km.

Refer to the above sketch:

$\angle ACX = 180° - \theta - 90° = 90° - \theta$

Hence, $\angle ECD = 90° - (90° - \theta) = \theta$

$\sin\theta = \frac{XC}{30}$ $\qquad$ $\cos\theta = \frac{EC}{20}$

$XC = 30\sin\theta$ $\qquad$ $EC = 20\cos\theta$

Shortest distance of D from the border $= YD$

$= XC - EC$

$= (30\sin\theta - 20\cos\theta)$ km

(b) Express $30\sin\theta - 20\cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° \leqslant \alpha \leqslant 90°$. [4]

$R = \sqrt{30^2 + 20^2}$

$= \sqrt{1300}$

$= 10\sqrt{13}$

$\alpha = \tan^{-1}\frac{20}{30}$

$= 33.690$

$= 33.7°$

$\therefore 30\sin\theta - 20\cos\theta = \underline{10\sqrt{13}\sin(\theta - 33.7°)}$

(c) Find the value of θ if village D is 10 km from the border. [2]

Let $10\sqrt{13}\sin(\theta - 33.7°) = 10$

$\sin(\theta - 33.7°) = \frac{1}{\sqrt{13}}$

Step 1: Basic angle for $\theta - 33.7° = 16.1°$

Step 2: $\theta - 33.7°$ lies in 1st or 2nd quad.

Step 3: $\theta - 33.7° = 16.1°$ or $\theta - 33.7° = 180° - 16.1°$

$\underline{\theta = 49.8°}$ or $\theta = 197.6°$

(rej. as θ is acute)

8 The equation of a curve is $y = (x-3)\sqrt{2x+1}$.

(a) Show that $\frac{dy}{dx} = \frac{3x-2}{\sqrt{2x+1}}$. [4]

$$\frac{dy}{dx} = (x-3)\cdot\cancel{\tfrac{1}{2}}(2x+1)^{-\frac{1}{2}}\cdot(\cancel{2}) + (2x+1)^{\frac{1}{2}}\cdot(1)$$
$$= (2x+1)^{-\frac{1}{2}}\cdot\left[(x-3) + (2x+1)\right]$$
$$= (2x+1)^{-\frac{1}{2}}(3x-2)$$
$$= \underline{\frac{3x-2}{\sqrt{2x+1}}} \quad \text{(shown)}$$

(b) A particle moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.15 units per second. Find the rate of change of the y-coordinate at the point where $x = 4$. [2]

Given $\frac{dx}{dt} = 0.15$

Find $\frac{dy}{dt}$ when $x = 4$.

Chain-rule : $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$$= \frac{3(4)-2}{\sqrt{2(4)+1}} \times 0.15$$
$$= \underline{0.5 \text{ units/s}}$$

(c) Use the result from **part (a)** to evaluate $\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx$. [4]

$$\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx = \frac{1}{3}\int_0^4 \frac{3x}{\sqrt{2x+1}}\,dx$$

$$= \frac{1}{3}\int_0^4 \frac{(3x-2)+2}{\sqrt{2x+1}}\,dx$$

$$= \frac{1}{3}\int_0^4 \frac{3x-2}{\sqrt{2x+1}}\,dx + \frac{2}{3}\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$$

$$= \frac{1}{3}\Big[(x-3)\sqrt{2x+1}\Big]_0^4 + \frac{2}{3}\int_0^4 (2x+1)^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{3}\big[3-(-3)\big] + \frac{2}{3}\left[\frac{(2x+1)^{\frac{1}{2}}}{(\frac{1}{2})(2)}\right]_0^4$$

$$= 2 + \frac{2}{3}\Big[\sqrt{2x+1}\Big]_0^4$$

$$= 2 + \frac{2}{3}(3-1)$$

$$= \underline{\underline{\frac{10}{3}}}$$

9 The equation of a circle is $x^2+y^2-16x-2y+40=0$.

(a) Find the radius and coordinates of the centre of the circle. [4]

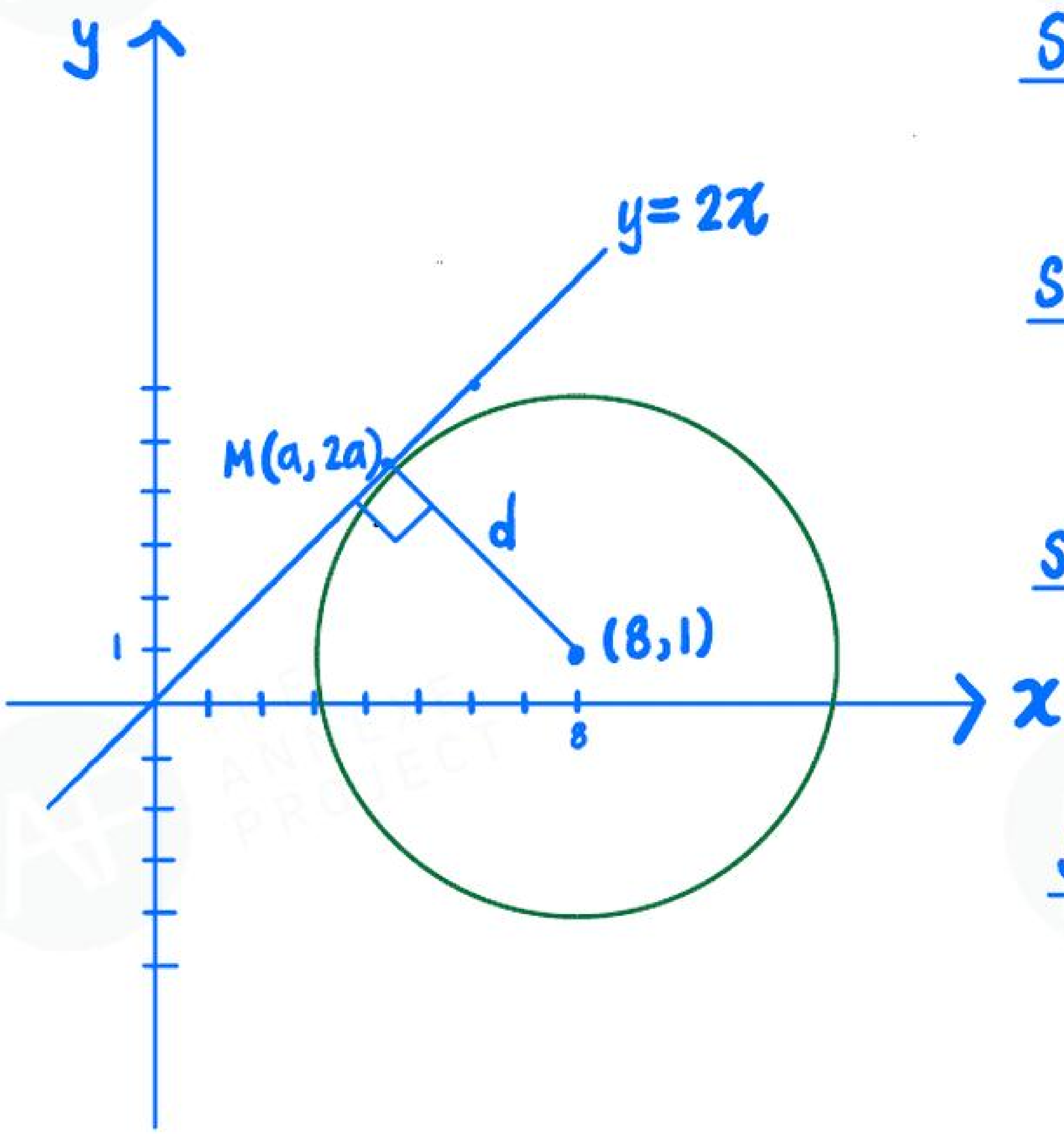
$(x^2-16x)+(y^2-2y)+40=0$

$(x-8)^2-8^2+(y-1)^2-1^2+40=0$

$(x-8)^2+(y-1)^2=5^2$

radius = 5 units, centre (8, 1)

(b) Find the shortest distance of the centre of the circle from the line $y=2x$ and hence explain why the circle does not intersect the line $y=2x$. [5]



Step 1: Let the shortest distance of centre of circle to $y=2x$ be d.

Step 2: the line d is perpendicular to $y=2x$, i.e. has a gradient of $\frac{-1}{2}$.

Step 3: Let the intersection of line d and $y=2x$ be M. Let x-coordinate of M = a, then y-coordinate of M = 2a.

Step 4: $\frac{2a-1}{a-8}=-\frac{1}{2}$

$8-a=4a-2$

$5a=10$

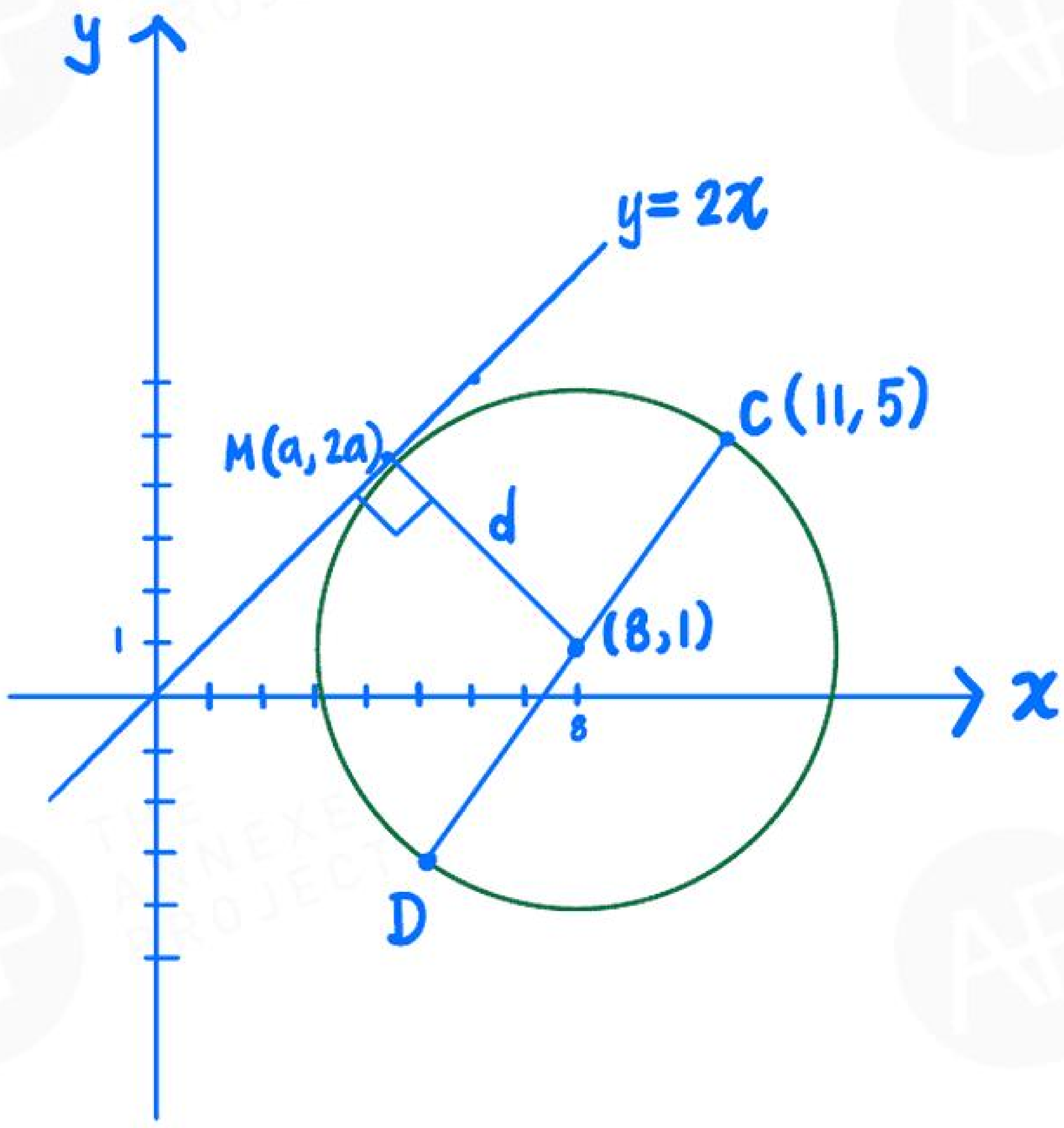
$a=2$ $\quad\therefore M=(2,4)$

Step 5: $d=\sqrt{(8-2)^2+(1-4)^2}$

$=\sqrt{45}$

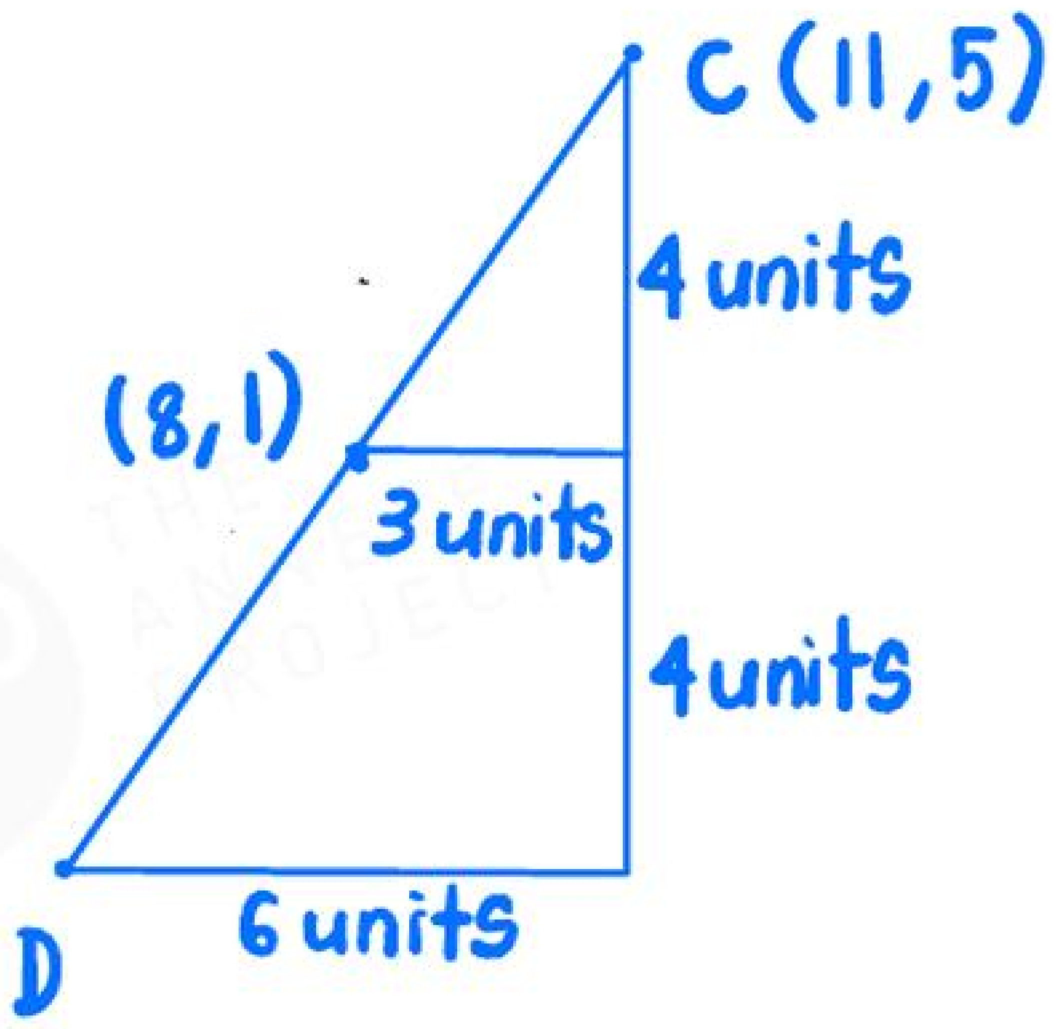
$=3\sqrt{5}$ units / 6.71

Since d > radius of circle, the circle does not intersect $y=2x$.



The point C (11, 5) lies on the circle and CD is a diameter of the circle.

(c) Find the coordinates of D. [2]

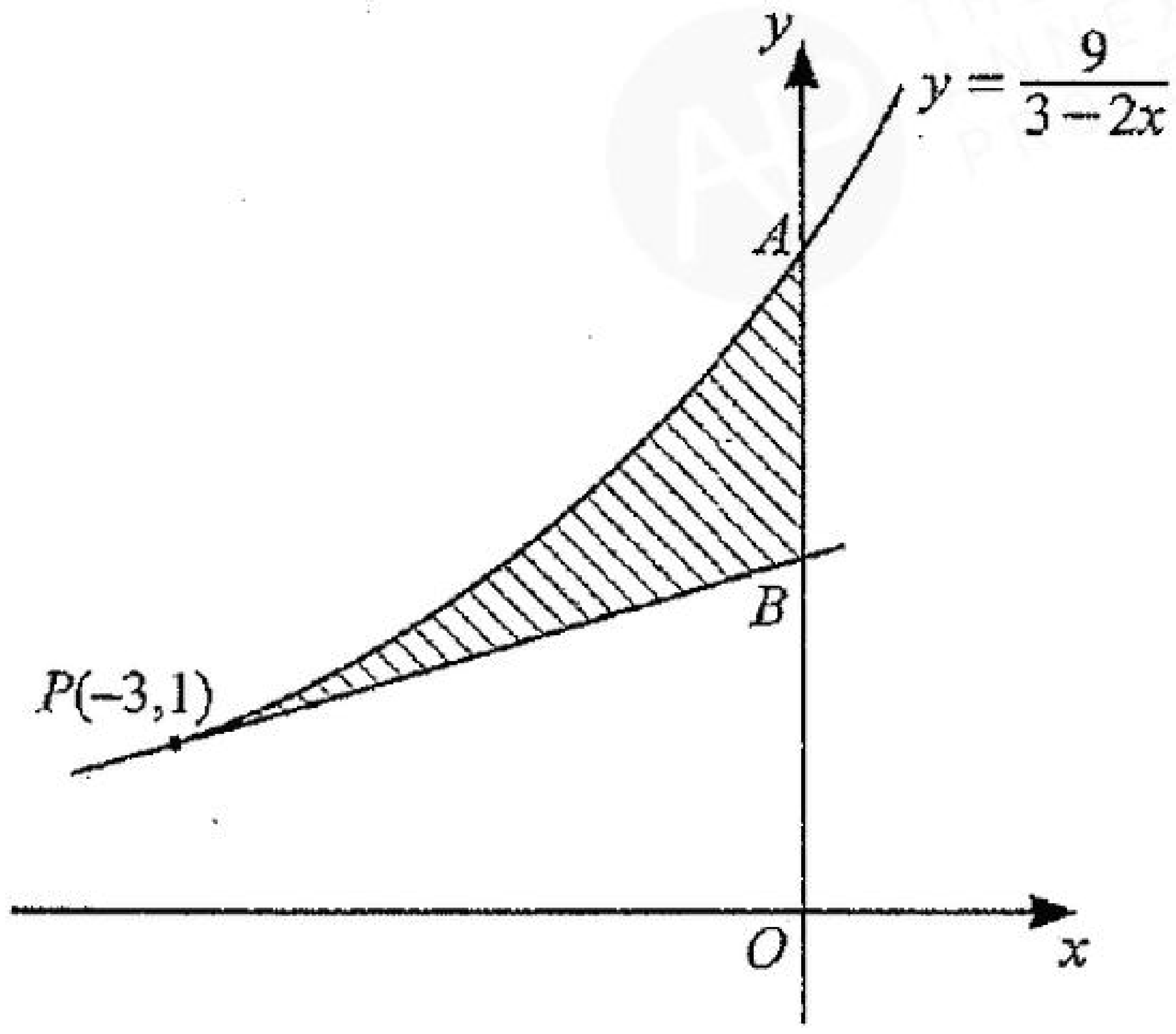


By similar $\triangle$s

x-coordinate of $D = 11 - 6$
$= 5$

y-coordinate of $D = 5 - 8$
$= -3$

$\therefore$ $\underline{D = (5, -3)}$



The diagram shows part of the curve $y = \frac{9}{3-2x}$ and the tangent to the curve at the point $P(-3, 1)$. The curve intersects the y-axis at A and the tangent intersects the y-axis at B.

(a) Determine, with full working, whether B is nearer to O or to A. [6]

Point A: Sub $x = 0$ into $y = \frac{9}{3-2x}$

$y = \frac{9}{3}$

$= 3$ $\quad \therefore A = (0, 3)$

Finding Equation of tangent line:

$y = 9(3-2x)^{-1}$

$\frac{dy}{dx} = 9(-1)(3-2x)^{-2} \cdot (-2)$

$= \frac{18}{(3-2x)^2}$

When $x = -3$, gradient of tangent line $= \frac{18}{(3+6)^2}$

$= \frac{2}{9}$

Equation of tangent line: $y - 1 = \frac{2}{9}(x+3)$

$y = \frac{2}{9}x + \frac{5}{3}$

Point B: Sub $x = 0$ into $y = \frac{2}{9}x + \frac{5}{3}$

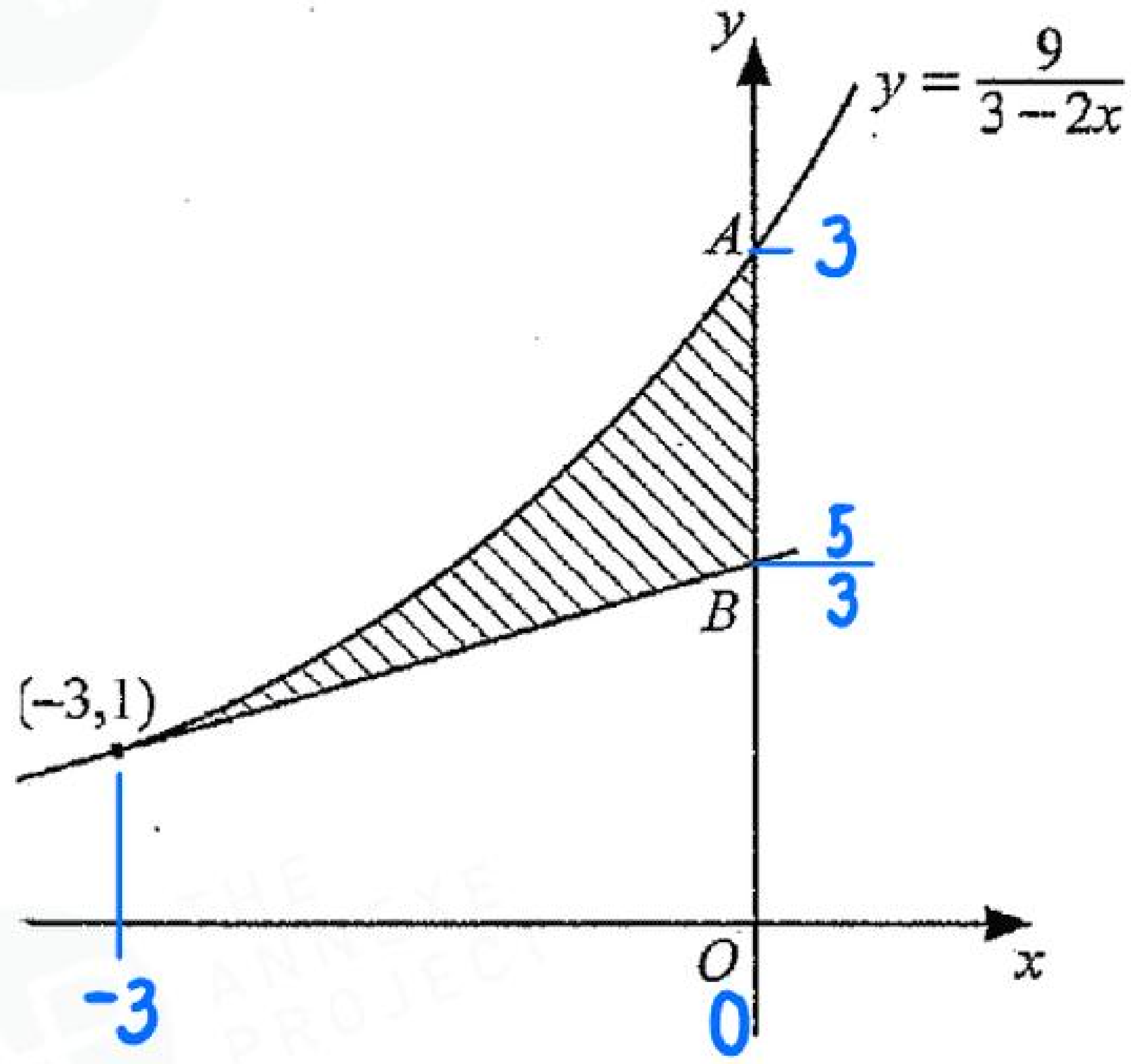
$y = \frac{5}{3}$ $\quad \therefore B = (0, \frac{5}{3})$

distance of OB $= \frac{5}{3}$ units.

distance of AB $= 3 - \frac{5}{3} = \frac{4}{3}$ units.

$\therefore$ B is nearer to A.

(b) Find, showing all necessary working, the value of the constants a and b for which the area of the shaded region can be expressed as $a\ln 3 + b$. [6]



Shaded Area

= Area under Curve − Area of Trapezium

$= \int_{-3}^{0} \frac{9}{3-2x}\,dx - \left[\frac{3}{2}\left(1+\frac{5}{3}\right)\right]$

$= \frac{9}{-2}\left[\ln(3-2x)\right]_{-3}^{0} - 4$

$= \frac{-9}{2}(\ln 3 - \ln 9) - 4$

$= \frac{-9}{2}(\ln 3 - 2\ln 3) - 4$

$= \left(\frac{9}{2}\ln 3 - 4\right)$ units^2.

where $a = \frac{9}{2}$, $b = -4$

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