CANDIDATE NAME

CENTRE NUMBER S ☐ ☐ ☐ ☐

INDEX NUMBER ☐ ☐ ☐ ☐

# MATHEMATICS

**4052/02**

Paper 2

October/November 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE ON ANY BARCODES.

Answer all questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142.

1  (a)  The table shows the resident population of Singapore in three years.

| Year | 2000 | 2010 | 2020 |
|---|---|---|---|
| Resident population | 3 273 363 | 3 771 721 | 4 044 210 |

(i) Write the 2010 resident population in standard form, correct to three significant figures.

Answer  $3.77 \times 10^6$  [1]

(ii) Calculate the percentage increase in the resident population from 2000 to 2020.

$$\frac{4044210 - 3273363}{3273363} \times 100\% = 23.5\%$$

Answer  23.5  % [2]

(iii) In 2000, 18.7% of the total population of Singapore was non-resident.

Calculate the total population of Singapore in 2000.

$$81.3\% = 3273363$$
$$100\% = \frac{3273363}{81.3} \times 100 = 4026276.753$$
$$= 4026277$$

Answer  4026 277  [2]

(b) Population density is defined as the number of people per square kilometre.
In 2020, the population density of Singapore was 7810 people per square kilometre.
In 2020, the total population of Singapore was 5.69 million.

Calculate the area of Singapore.

$$\frac{5690000}{7810} = 728.55$$
$$= 729 \text{ km}^2$$

Answer  729  km$^2$ [2]

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**The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.**

(c) The diameter of a red blood cell is $7.8 \times 10^{-6}$ m.
In a scale drawing, the diameter of a red blood cell is 3.9 cm.

(i) Find the scale used for the drawing.
Give your answer in the form $n : 1$.

3.9 cm represents $7.8 \times 10^{-6}$ m

divide both sides by $7.8 \times 10^{-6}$

0.039 m represents $7.8 \times 10^{-6}$

5000 : 1

Answer ........5000........ : 1 [2]

(ii) Drawn to the same scale as the red blood cell, the diameter of a blood platelet is 8 mm.

Find the actual diameter of the blood platelet.
Give your answer in millimetres in standard form.

$\dfrac{8 \, mm}{5000} = 0.0016$ mm
$= 1.6 \times 10^{-3}$ mm.

Answer ........$1.6 \times 10^{-3}$........ mm [1]

2   (a)  Solve $6 - 7x = 5(1 - 2x)$.

$$6 - 7x = 5 - 10x$$
$$3x = -1$$
$$x = \frac{-1}{3}$$

Answer  $x = $ ................$\frac{-1}{3}$................ [2]

(b)  Solve the inequality $5 - y > 7$.

$$5 - 7 > y$$
$$y < -2$$

Answer  ................$y < -2$................ [1]

(c)  $c = \dfrac{1}{b} + \dfrac{a}{3-a}$

  (i)  Find $c$ when $a = 2$ and $b = 4$.

$$c = \frac{1}{4} + \frac{2}{3-2}$$
$$= \frac{1}{4} + 2$$
$$= \frac{9}{4}$$

Answer  $c = $ ................$\frac{9}{4}$................ [1]

  (ii)  Rearrange the formula to make $a$ the subject.

$$c - \frac{1}{b} = \frac{a}{3-a}$$
$$\frac{bc - 1}{b} = \frac{a}{3-a}$$
$$(bc - 1)(3 - a) = ab$$
$$3bc - abc - 3 + a = ab$$
$$a - ab - abc = 3 - 3bc$$
$$a(1 - b - bc) = 3(1 - bc)$$
$$a = \frac{3(1 - bc)}{(1 - b - bc)}$$

Answer  $a = $ ................$\dfrac{3(1-bc)}{(1-b-bc)}$................ [3]

**The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.**

**(d)** Solve the equation $\dfrac{x}{2x-1} - \dfrac{6}{4-x} = 3$.

Give your solutions correct to two decimal places.

$$\frac{x(4-x) - 6(2x-1)}{(2x-1)(4-x)} = 3$$

$$4x - x^2 - 12x + 6 = 3(2x-1)(4-x)$$

$$-x^2 - 8x + 6 = 3(-2x^2 + 9x - 4)$$

$$5x^2 - 35x + 18 = 0$$

$$x = \frac{-(-35) \pm \sqrt{(-35)^2 - 4(5)(18)}}{2(5)}$$

$$= \frac{35 \pm \sqrt{865}}{10}$$
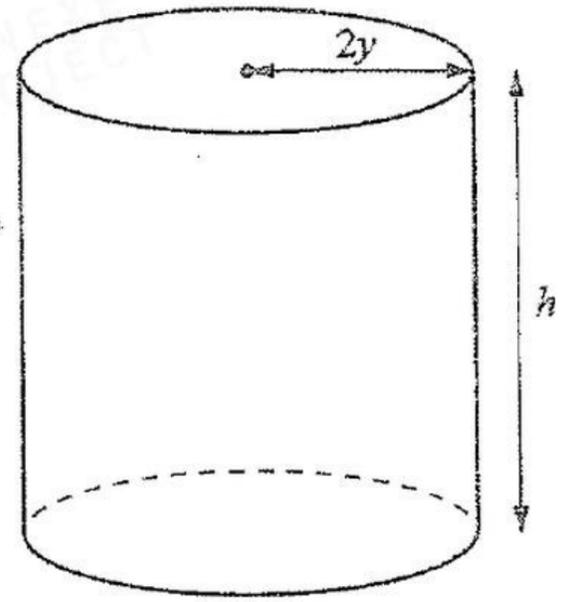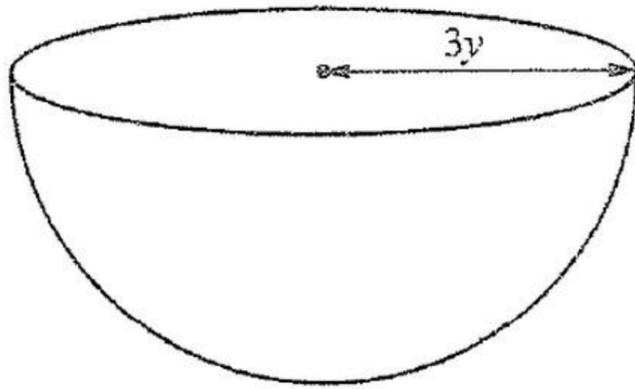
$$= 0.56 \text{ or } 6.44$$

*Answer*  $x = \underline{\;0.56\;}$ or $\underline{\;6.44\;}$ [5]

The diagram shows a solid hemisphere and a solid cylinder.
The hemisphere has radius $3y$ cm.
The cylinder has radius $2y$ cm and height $h$ cm.

**(a)** Show that the total surface area of the hemisphere is $27\pi y^2$ cm$^2$.

*Answer*

$$\pi(3y)^2 + 2\pi(3y)^2$$
$$= 9\pi y^2 + 18\pi y^2$$
$$= 27\pi y^2 \text{ cm}^2$$

Curved surface area
of sphere $= 4\pi r^2$

Curved surface area
of hemisphere $= 2\pi r^2$

[2]

**(b)** The total surface area of the cylinder is equal to the total surface area of the hemisphere.

Find $h$ in terms of $y$.

$$2\pi(2y)^2 + 2\pi(2y)h = 27\pi y^2$$
$$8y^2 + 4hy = 27y^2$$
$$4hy = 19y^2$$
$$h = \frac{19y}{4}$$

*Answer* $h = \underline{\quad \frac{19}{4}y \quad}$ [3]

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

**(c)** The volume of the hemisphere is $500\,\text{cm}^3$.

Calculate the volume of the cylinder.

$$\frac{2}{3}\pi(3y)^3 = 500$$

$$18\pi y^3 = 500$$

$$y = \sqrt[3]{\frac{500}{18\pi}}$$

$$= 2.0678 \text{ cm}$$

$$\therefore h = \frac{19}{4}(2.0678)$$

$$= 9.8222 \text{ cm}$$

$$\text{Vol. of cylinder} = \pi(2y)^2 h$$

$$= 4\pi y^2 h$$

$$= 4\pi(2.0678)^2(9.8222)$$

$$= 527.78$$

$$= 528 \text{ cm}^3$$

Answer ................**528**................ $\text{cm}^3$ [4]

4  (a)  Complete the table of values for $y = \dfrac{2}{x^2} + 3x - 1$.

Values are given to one decimal place where appropriate.

**When $x = -3$, $y = \dfrac{2}{(-3)^2} + 3(-3) - 1 = -9.8$**

| $x$ | $-3$ | $-2$ | $-1$ | $-0.5$ | $0.5$ | $1$ | $1.5$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-9.8$ | $-6.5$ | $-2$ | $5.5$ | $8.5$ | $4$ | $4.4$ | $5.5$ | $8.2$ |

[1]

(b)  On the grid opposite, draw the graph of $y = \dfrac{2}{x^2} + 3x - 1$ for $-3 \leqslant x \leqslant 3$.  [3]

(c)  (i)  On the same grid, draw the graph of $2y - 5x = 7$ for $-3 \leqslant x \leqslant 3$.

[2]

(ii)  Write down the $x$-coordinates of the points where the line intersects the curve.

*Answer*  $x = $ ....**$-0.6$**.... and ....**$0.7$**.... [2]

(iii)  These values of $x$ are solutions of the equation $x^3 + Ax^2 + B = 0$.

Find the value of $A$ and the value of $B$.

**$\underline{\text{Sub } y = \dfrac{2}{x^2} + 3x - 1 \text{ into } 2y - 5x = 7:}$**

**$2\left(\dfrac{2}{x^2} + 3x - 1\right) - 5x = 7$**

**$\dfrac{4}{x^2} + 6x - 2 - 5x - 7 = 0$**

**$\dfrac{4}{x^2} + x - 9 = 0$**

**$x^3 - 9x^2 + 4 = 0$**

*Answer*  $A = $ ........**$-9$**........

$B = $ ........**$4$**........

[3]

$y = \dfrac{2}{x^2} + 3x - 1$

| $x$ | $-3$ | $-2$ | $-1$ | $-0.5$ | $0.5$ | $1$ | $1.5$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | **-9.8** | $-6.5$ | $-2$ | $5.5$ | $8.5$ | $4$ | $4.4$ | $5.5$ | $8.2$ |



$2y - 5x = 7$

| $x$ | $-3$ | $0$ | $3$ |
|---|---|---|---|
| $y$ | $-4$ | $3.5$ | $11$ |

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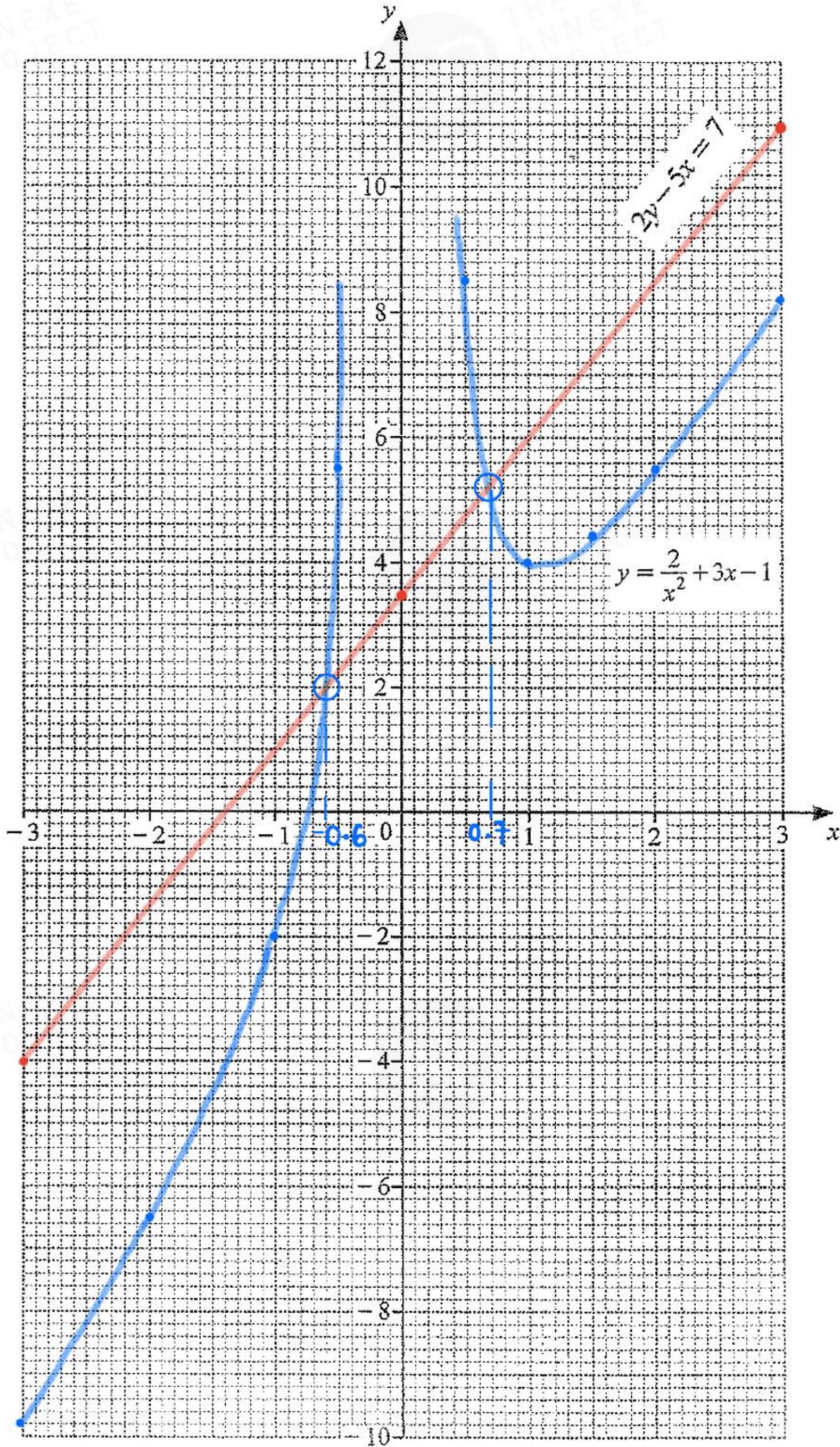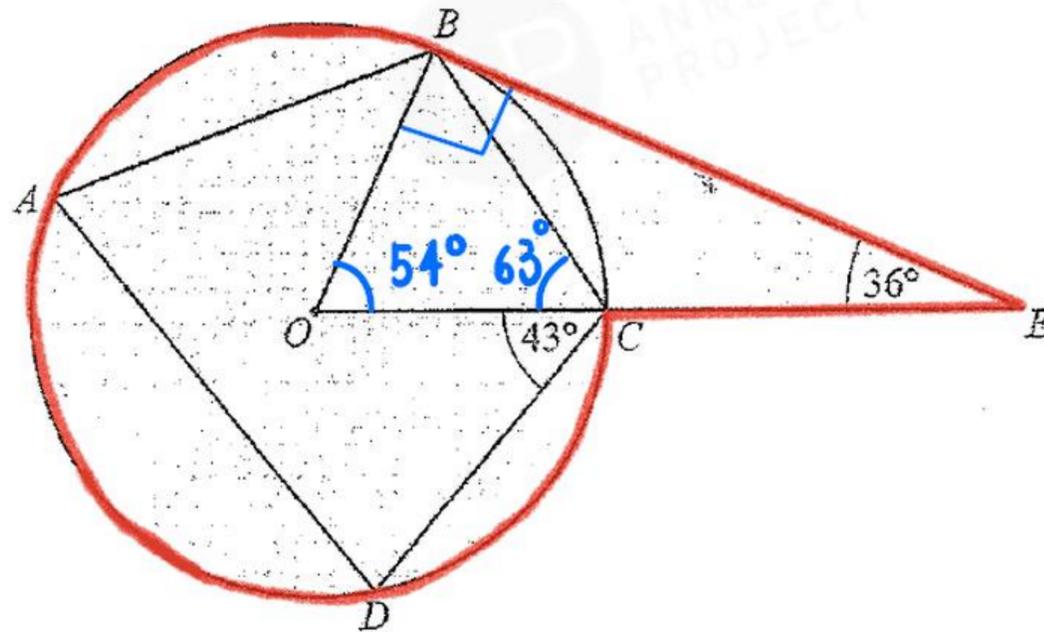
$A$, $B$, $C$ and $D$ are points on the circle, centre $O$.
$EB$ is a tangent to the circle at $B$ and $OE$ is a straight line that passes through point $C$.
Angle $OEB = 36°$ and angle $OCD = 43°$.

(a) Find angle $BAD$.
    Give reasons for each step of your working.

$\angle OBE = 90°$ ( tan $\perp$ radius)

$\therefore \angle BOE = 180° - 90° - 36°$ ( sum of $\triangle$ )

$\qquad = 54°$

$\angle OCB = \dfrac{180° - 54°}{2}$ ($OB = OC$, base $\angle$s of isos. $\triangle$)

$\qquad = 63°$

$\angle BAD = 180° - (63° + 43°)$ ($\angle$s in opp. segment)

$\qquad = 74°$

*Answer* Angle $BAD = $ ..........74°.......... [4]

**(b)** The radius of the circle is 8 cm.

Calculate the perimeter of the shaded shape.

$$\text{Length of major arc } BADC = \frac{360° - 54°}{360°} \times 2\pi(8)$$
$$= 42.726 \text{ cm.}$$

$$\tan 36° = \frac{8}{BE}$$
$$\therefore BE = \frac{8}{\tan 36°} = 11.011 \text{ cm.}$$

$$\sin 36° = \frac{8}{OE}$$
$$\therefore OE = \frac{8}{\sin 36°} = 13.610 \text{ cm.}$$

Hence $CE = 13.610 - 8 = 5.61 \text{ cm.}$

$$P = 42.726 + 11.011 + 5.61$$
$$= 59.3 \text{ cm}$$

*Answer* ................**59.3**................ cm [4]

6   (a)   $p = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ and $q = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

$X$ is the point $(7, 3)$ and $Y$ is the point $(3, 4)$.
$\overrightarrow{XY} = mp + nq$, where $m$ and $n$ are constants.

Find the value of $m$ and the value of $n$.

$$\overrightarrow{XY} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ 1 \end{pmatrix} = m \begin{pmatrix} -4 \\ 5 \end{pmatrix} + n \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$-4m + 2n = -4$$
$$n = 2m - 2 \quad \text{(1)}$$
$$5m - 3n = 1 \quad \text{(2)}$$

Sub (1) into (2):

$$5m - 3(2m - 2) = 1$$
$$-m + 6 = 1$$
$$\underline{m = 5}$$
$$\therefore n = 2(5) - 2$$
$$= \underline{8}$$

Answer   $m = $ ........5........

$n = $ ........8........

[4]

**(b)** $A$ is the point $(4, 6)$, $B$ is the point $(-5, -3)$ and $C$ is the point $(8, -4)$.
$D$ is the point $(-1, y)$ which lies on the line $AB$.

**(i)** Find the value of $y$.

$$\text{grad} \cdot AB = \text{grad} \cdot AD$$

$$\frac{-3-6}{-5-4} = \frac{y-6}{-1-4}$$

$$\therefore \frac{y-6}{-5} = 1$$

$$y-6 = -5$$

Answer $y = $ .......1............... [1]

**(ii)** Find the length of the line $DC$.

$$\text{Length } DC = \sqrt{(8+1)^2 + (-4-1)^2}$$

$$= \sqrt{81 + 25}$$

$$= 10.3$$

Answer ...........10.3........... [2]

**(iii)** $C$ is the point on the line $AE$ such that $AC : AE = 2 : 3$.

Find the equation of the line $BE$.

A (4,6)

10 units

2

4 units

C (8,-4)

5 units

1

E

2 units
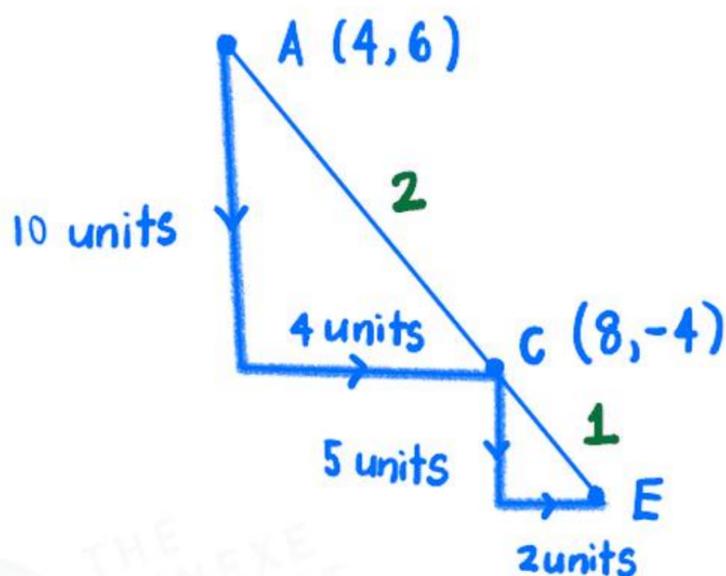
Equation of line BE:

$$y-(-9) = \frac{-2}{5}(x-10)$$

$$y+9 = -\frac{2}{5}x + 4$$

$$y = -\frac{2}{5}x - 5$$

By similar $\triangle$s:

x-coordinates of $E = 8+2 = 10$
y-coordinates of $E = -4-5 = -9$
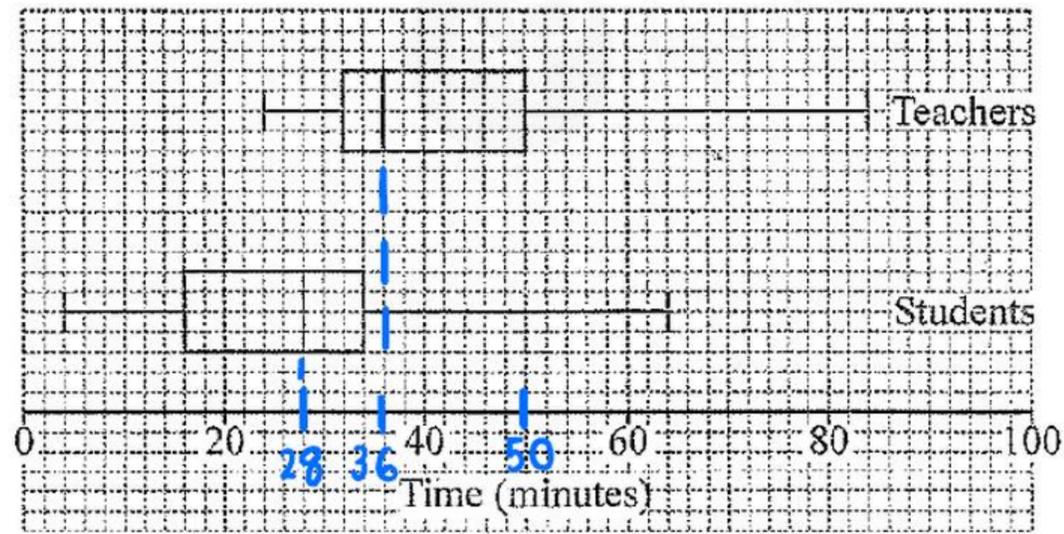
$\therefore E(10, -9)$

gradient of $BE = \frac{-3-(-9)}{-5-10} = \frac{6}{-15} = \frac{-2}{5}$

Answer $y = -\frac{2}{5}x - 5$ .............. [4]

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7   (a)  The students and teachers at a school record the time it took them to travel to school.
         The box-and-whisker plots show the distributions of the results.



(i)   There are 18 teachers who took more than 50 minutes to travel to school.

      Work out the number of teachers in the school.

      **Upper Quartile for teachers = 18   (Top 25%)**
      **Hence, total no. of teachers**          *Answer* .................**72**................. [1]
      **= 4 × 18 = 72**

(ii)  Make a comment comparing the averages and a comment comparing the distributions of the
      times taken by the students and the teachers.
      Use figures to support your answers.

      1. **Students took less time on average to travel to school,**
         **as the median time for students (28 mins) is less than**
         **the median time for teachers (36 mins).**
      2. **The travelling time for both teachers and students are**
         **equally consistent as their interquartile range are**
         **both at 18 mins.**                                                            [3]

(b) In a class of 24 students, 8 walk to school, 6 cycle to school and the rest travel to school by bus.

(i) One student is selected from the class at random.

Find the probability that the student travels to school by bus.

$$\frac{24 - 8 - 6}{24} = \frac{5}{12}$$

Answer ................ $\frac{5}{12}$ ................ [1]

(ii) Two students are selected from the class at random.

Find the probability that they both use the same type of transport to travel to school.

$$\{W, W\} + \{C, C\} + \{B, B\}$$

$$= \left(\frac{8}{24} \times \frac{7}{23}\right) + \left(\frac{6}{24} \times \frac{5}{23}\right) + \left(\frac{10}{24} \times \frac{9}{23}\right)$$

$$= \frac{22}{69}$$

Answer ................ $\frac{22}{69}$ ................ [2]

(iii) Three of the students are selected from the class at random.

Find the probability that two of them walk to school and the other does not.

$$\left\{W, W, \begin{smallmatrix}\text{does not}\\\text{walk}\end{smallmatrix}\right\} + \left\{W, \begin{smallmatrix}\text{does not}\\\text{walk}\end{smallmatrix}, W\right\} + \left\{\begin{smallmatrix}\text{does not}\\\text{walk}\end{smallmatrix}, W, W\right\}$$

$$= \left(\frac{8}{24} \times \frac{7}{23} \times \frac{16}{22}\right) + \left(\frac{8}{24} \times \frac{16}{23} \times \frac{7}{22}\right) + \left(\frac{16}{24} \times \frac{8}{23} \times \frac{7}{22}\right)$$

$$= \frac{56}{253}$$

Answer ................ $\frac{56}{253}$ ................ [2]

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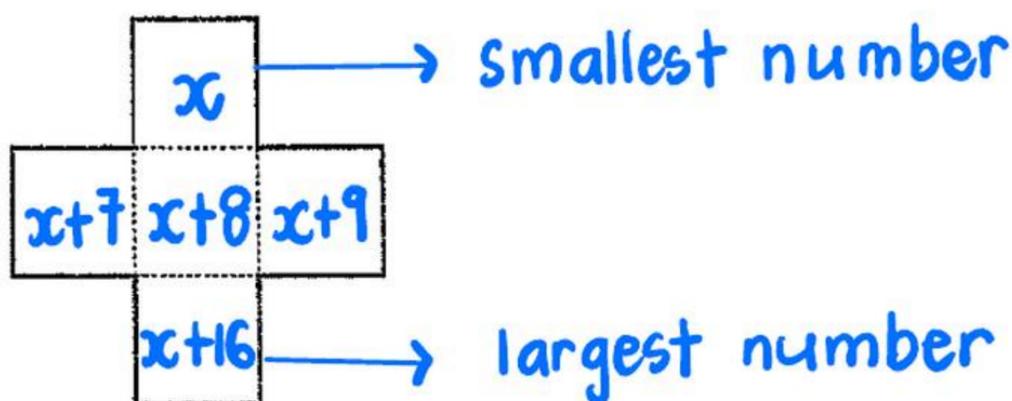4052/02/O/N/23

8  (a)  The diagram shows part of a number grid.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 |

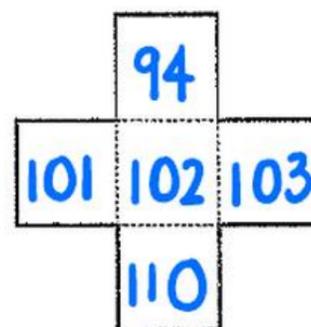A cross outlining five numbers, as shown, is placed on the grid.

(i)  The cross is placed in a different position on the grid.
The sum of the largest and smallest of the numbers outlined is 204.

Complete the cross to show the numbers outlined.

$$x + (x + 16) = 204$$
$$2x = 204 - 16$$
$$x = 94$$

*Answer*



x → smallest number

x+7  x+8  x+9

x+16 → largest number

94

101  102  103

110

[3]

(ii)  The cross can be placed anywhere on the grid.
The difference between the product of the left and right numbers and the product of the top and bottom numbers is found.

Show that this difference is always 63.

*Answer*

• product of the left and right numbers $= (x+7)(x+9)$
$$= x^2 + 16x + 63$$

product of the top and bottom numbers $= x(x+16)$
$$= x^2 + 16x$$

$$(x^2 + 16x + 63) - (x^2 + 16x) = 63 \text{ (shown)}.$$

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[3]

**(b)** The $n$th term of a sequence is given by $T_n = \dfrac{n^2 - 1}{3n - 1}$.

The term $T_k = \dfrac{15}{4}$.

Find the value of $T_{k+1}$.

$$\frac{15}{4} = \frac{K^2 - 1}{3K - 1}$$

$$45K - 15 = 4K^2 - 4$$

$$4K^2 - 45K + 11 = 0$$

$$(4K - 1)(K - 11) = 0$$

$$\therefore K = \frac{1}{4} \text{ (rej. because K has to}$$
$$\text{be an integer)}$$

$$\text{or } K = 11$$

$$\therefore T_{K+1} = T_{12} = \frac{12^2 - 1}{3(12) - 1}$$

$$= \frac{143}{35}$$

Answer $T_{k+1} = $ ..........$\dfrac{143}{35}$.......... [4]

**9** Chen is considering installing solar panels at his home.
He wants to estimate the area of solar panels he will need to generate the amount of electricity used by his household.
He finds this information on the internet.

| |
| --- |
| • Find the average amount of electricity, measured in kilowatt hours (kWh), used in one day |
| • Find the number of peak sunlight hours you receive per day |
| • Solar panel output in daily watt hours = solar panel watts × peak sunlight hours × 75% |
| • All solar panels should be of the same type |
| |
| • 1 kWh is the amount of energy used in 1 hour by an appliance with a power rating of 1 kW |
| • 1 kW = 1000 W |

He does some more research about peak sunlight hours and the specifications of common solar panels.

Average daily peak sunlight hours in Singapore by month



Peak sunlight hours vs Month
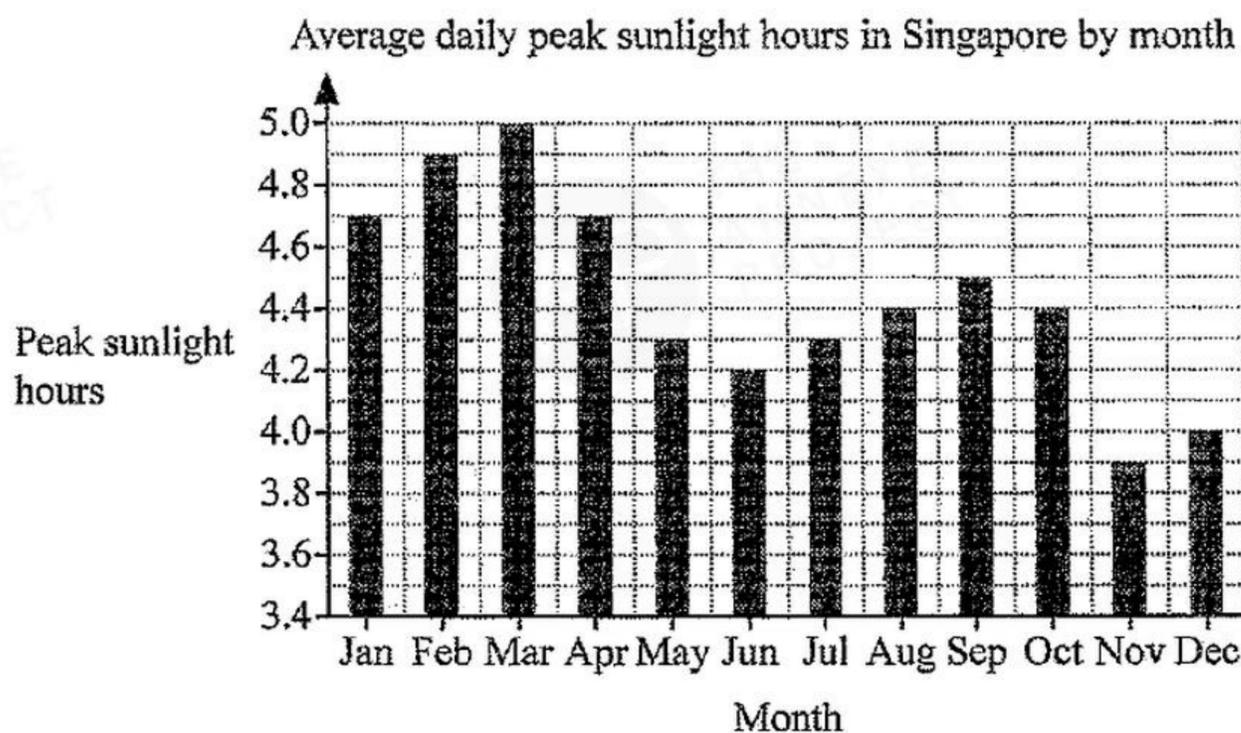
| Type | Solar panel wattage | Length of panel | Width of panel | Mass of panel |
| --- | --- | --- | --- | --- |
| A | 300 W | 1.46 m | 1.05 m | 18 kg |
| B | 370 W | 1.96 m | 1.00 m | 23 kg |
| C | 390 W | 1.98 m | 1.00 m | 28 kg |
| D | 400 W | 1.69 m | 1.05 m | 19 kg |

**(a)** A games console has a power rating of 165 W.

Calculate the amount of energy, in kWh, used in 5 hours by the games console.

$0.165 \text{ kW} \times 5h = 0.825 \text{ kWh}$

Answer ............ 0.825 ............ kWh [1]

**(b)** Assuming all months have an equal number of days, calculate the average daily peak sunlight hours in Singapore over the 12-month period.
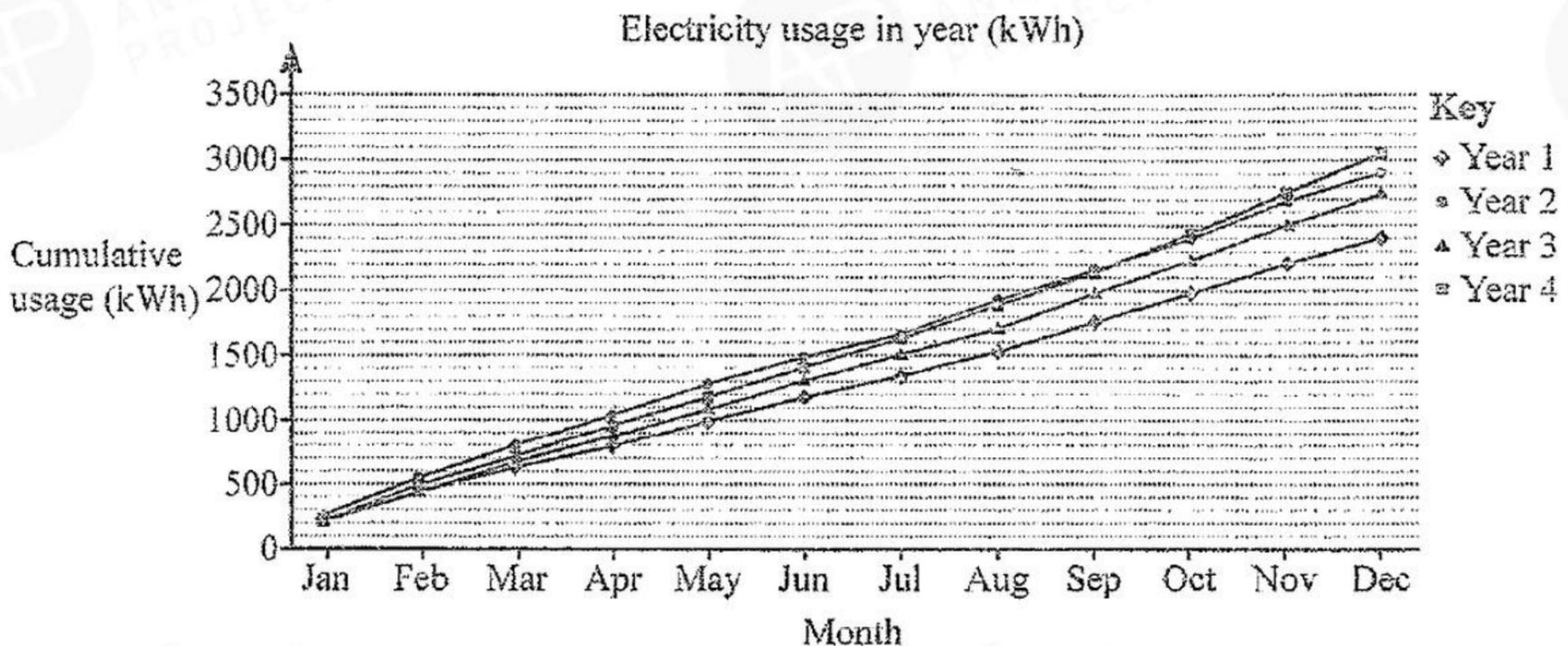
$$\frac{4.7 + 4.9 + 5 + 4.7 + 4.3 + 4.2 + 4.3 + 4.4 + 4.5 + 4.4 + 3.9 + 4}{12}$$

$= 4.4417$

$= 4.44$

Answer ............ 4.44 ............ hours [2]

(e) This graph shows the electricity used by Chen's household for the past four years.

### Electricity usage in year (kWh)



Find an estimate of the total area of solar panels Chen will need for his home.
Justify any decisions you make and show your calculations clearly.

Step 1 : Type A wattage = $\dfrac{300W}{1.46 \times 1.05\,m^2}$ = 196 W/m²

Type B wattage = $\dfrac{370W}{1.96 \times 1\,m^2}$ = 189 W/m²

Type C wattage = $\dfrac{390W}{1.98 \times 1\,m^2}$ = 197 W/m²

Type D wattage = $\dfrac{400W}{1.69 \times 1.05\,m^2}$ = 225 W/m²

Chen should use <u>type D solar panels</u> as it's most efficient.

Step 2 : Year 4 has the highest annual electricity usage of approx. 3050 kWh.

∴ daily electricity usage = $\dfrac{3050\,kWh}{365}$

= 8.3562 kWh.

≈ 8.36 kWh.

Step 3 :

Solar panel output in daily watt hours = solar panel watts × peak sunlight hours × 75%

= 400W × 4.4417h × 0.75
(type D)

= 1332.51Wh

= 1.3325 KWh

[7]

<u>Step 4</u> : No. of solar panels needed $= \dfrac{8 \cdot 3562}{1 \cdot 3325}$

$= 6 \cdot 2710$

$\approx 7$

$7 \times 1 \cdot 69 \times 1 \cdot 05$

$= 12 \cdot 4215$

$= \underline{12 \cdot 4 \ m^2}$