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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/02

Paper 2

October/November 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The manager of a coffee shop purchases a coffee machine for \$1800. The value of the machine, \$$V$, after a period of t months can be modelled by the formula $V = 1800e^{-kt}$ where k is a constant. After 12 months, the value of the machine has dropped to \$1000. The manager purchases a new machine when the value of the first machine is 25% of its purchase price. Find the value of t when a new machine is purchased. [4]

$$1000 = 1800e^{-k(12)}$$

$$\frac{5}{9} = e^{-12k}$$

$$\ln\frac{5}{9} = -12k$$

$$\therefore k = \frac{-1}{12}\ln\frac{5}{9}$$

$$= \underline{0.048982}$$

Let $0.25 \times 1800 = 1800e^{-0.048982t}$

$$\frac{450}{1800} = e^{-0.048982t}$$

$$\ln\frac{1}{4} = -0.048982t$$

$$t = \underline{28.3 \text{ months}}$$

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2 Find the range of values of the constant k for which the line $y = kx$ intersects the curve $y = x^2 + 3kx + 2 - k$ at two distinct points. [5]

$y = kx$ —— (1)

$y = x^2 + 3kx + 2 - k$ —— (2)

Sub (1) into (2):

$kx = x^2 + 3kx + 2 - k$

$x^2 + 2kx + (2-k) = 0$

To intersect at 2 distinct points,

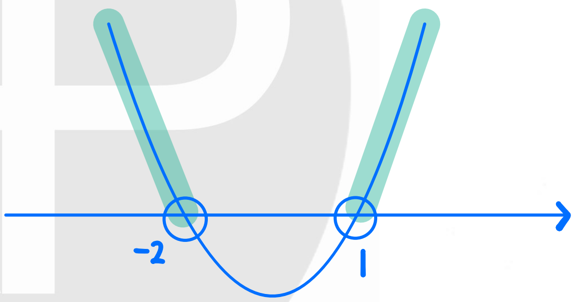
$b^2 - 4ac > 0$

$(2k)^2 - 4(2-k) > 0$

$4k^2 + 4k - 8 > 0$

$k^2 + k - 2 > 0$

$(k-1)(k+2) > 0$



$k < -2$ or $k > 1$

3 Solve the equation $\log_2 x - \log_4(x+3) = 3\log_8 2$. [5]

$$\log_2 x - \frac{\log_2(x+3)}{\log_2 4} = \log_8 2^3$$

$$\log_2 x - \frac{\log_2(x+3)}{\log_2 2^2} = 1$$

$$\log_2 x - \tfrac{1}{2}\log_2(x+3) = 1$$

$$\log_2 x - \log_2\sqrt{x+3} = 1$$

$$\log_2 \frac{x}{\sqrt{x+3}} = 1$$

$$\frac{x}{\sqrt{x+3}} = 2$$

$$x = 2\sqrt{x+3}$$

$$x^2 = 4(x+3)$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$\underline{x = 6}$ or $x = -2$ (Rej. because $\log_2 -2$ is undefined)

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4 (a) (i) Find the gradient of the curve $y = \frac{4x}{5-3x}$ at the point where $x = 1$. [3]

$$\frac{dy}{dx} = \frac{(5-3x)(4) - (4x)(-3)}{(5-3x)^2}$$

$$= \frac{20 - 12x + 12x}{(5-3x)^2}$$

$$= \frac{20}{(5-3x)^2}$$

When $x = 1$, $\frac{dy}{dx} = \frac{20}{4} = 5$

(ii) Explain why the curve has no stationary points. [1]

$$\frac{20}{(5-3x)^2} > 0$$

Hence, the curve has no stationary points.

(b) The curve $y = f(x)$ is such that $f'(x) = \frac{4}{\cos^2 x}$. The curve passes through the point $\left(\frac{\pi}{4}, 1\right)$. Find an expression for $f(x)$. [3]

$$\frac{dy}{dx} = \frac{4}{\cos^2 x} = 4\sec^2 x$$

$$\therefore y = \int 4\sec^2 x \; dx$$

$$= 4\tan x + c$$

Sub $\left(\frac{\pi}{4}, 1\right)$: $1 = 4\tan\frac{\pi}{4} + c$

$1 = 4 + c$

$c = -3$

$f(x) = 4\tan x - 3$

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5 The equation of a curve is $y = e^x + e^{-x}$.

(a) Verify that the curve has a stationary point at $x = 0$. [2]

$y = e^x + e^{-x}$

$\frac{dy}{dx} = e^x - e^{-x}$

$= e^x - \frac{1}{e^x}$

When $x = 0$, $\frac{dy}{dx} = e^0 - \frac{1}{e^0} = 1 - 1 = 0$

Hence, the curve has a stationary point at $x = 0$.

(b) Solve the equation $e^x + e^{-x} = 2.9$, giving values of x in logarithmic form. [4]

$e^x + \frac{1}{e^x} = 2.9$

Sub $u = e^x$: $u + \frac{1}{u} = 2.9$

$u^2 - 2.9u + 1 = 0$

$10u^2 - 29u + 10 = 0$

$(5u - 2)(2u - 5) = 0$

$\therefore u = \frac{2}{5}$ or $\frac{5}{2}$

$e^x = \frac{2}{5}$ or $e^x = \frac{5}{2}$

$x = \ln \frac{2}{5}$ or $\ln \frac{5}{2}$

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6 (a) The expressions $x^3+x^2+ax+2b$ and x^3+4x^2-ax+b, where a and b are constants, both have a factor of $x+3$. Find the values of a and b. [4]

Let $g(x) = x^3 + x^2 + ax + 2b$

$g(-3) = -27 + 9 - 3a + 2b = 0$

$\therefore 2b - 3a = 18$

$b = \frac{3}{2}a + 9$ — (1)

Let $h(x) = x^3 + 4x^2 - ax + b$

$h(-3) = -27 + 36 + 3a + b = 0$

$3a + b = -9$ — (2)

Sub (1) into (2):

$3a + \frac{3}{2}a + 9 = -9$

$4.5a = -18$

$a = -4$

$\therefore b = \frac{3}{2}(-4) + 9 = 3$

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(b) Solve the equatic

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(b) Solve the equation $x^3+2x^2-11x+6=0$, expressing the roots in an exact form. [5]

Step 1 :

Let $f(x) = x^3+2x^2-11x+6$

$f(2) = 8+8-22+6$

$= 0$

$\therefore (x-2)$ is a factor.

Step 2 :

$$
\begin{array}{r}
x^2+4x-3 \\
x-2 \overline{\smash{)}\, x^3+2x^2-11x+6} \\
-(x^3-2x^2) \\
\hline
4x^2-11x \\
-(4x^2-8x) \\
\hline
-3x+6 \\
-(-3x+6) \\
\hline
0
\end{array}
$$

Step 3 : let $x^2+4x-3=0$

$$x = \frac{-4 \pm \sqrt{4^2-4(-3)}}{2}$$

$$= \frac{-4 \pm \sqrt{28}}{2}$$

$$= \frac{-4 \pm 2\sqrt{7}}{2}$$

$$= -2 \pm \sqrt{7}$$

$x^3+2x^2-11x+6=0$

$(x-2)(x^2+4x-3)=0$

$x = -2-\sqrt{7}$, $-2+\sqrt{7}$ or 2

7 Points $P(3, 1)$ and $Q(-5, 7)$ lie on a circle and PQ is a diameter of the circle.

(a) Find the coordinates of the centre of the circle and the radius. [3]

$$\text{centre} = \left(\frac{3-5}{2}, \frac{1+7}{2}\right)$$
$$= (-1, 4)$$
$$\text{radius} = \sqrt{(3+1)^2+(1-4)^2}$$
$$= \underline{5 \text{ units}}$$

(b) Find the equation of the circle in the form $x^2+y^2+2gx+2fy+c=0$. [2]

$$(x+1)^2+(y-4)^2 = 25$$
$$x^2+2x+1+y^2-8y+16-25=0$$
$$x^2+y^2+2x-8y-8=0$$

where $\underline{g=1, \ f=-4, \ c=-8}$

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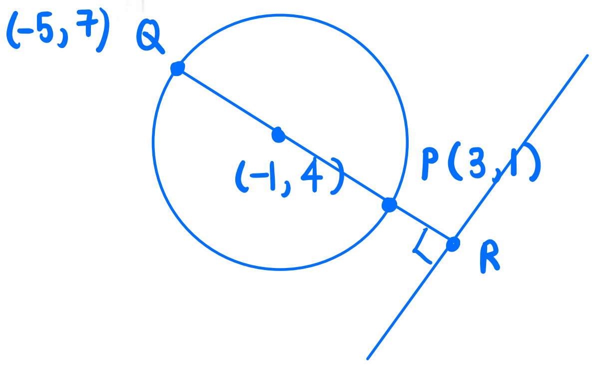
The point R lies on the line $3y = 4x - 34$ and is the point on the line that is closest to the circle.

(c) Without finding the coordinates of R, explain why R lies on QP extended. [1]



$3y = 4x - 34$

$\therefore y = \frac{4}{3}x - \frac{34}{3}$ ——— (1)

gradient $QP = \frac{7-1}{-5-3}$

$= \frac{6}{-8}$

$= -\frac{3}{4}$

$\therefore QP \perp 3y = 4x - 34$

(d) Hence find the coordinates of R. [3]

Equation of line QP:

$y - 7 = -\frac{3}{4}(x+5)$

$y = -\frac{3}{4}x + \frac{13}{4}$ ——— (2)

equating (1) and (2): $\frac{4}{3}x - \frac{34}{3} = -\frac{3}{4}x + \frac{13}{4}$

$\frac{25}{12}x = \frac{175}{12}$

$x = 7$

When $x = 7$, $y = -\frac{3}{4}(7) + \frac{13}{4}$

$= -2$

$\therefore R = (7, -2)$

8 At time $t = 0$, where t is measured in seconds, a racing car comes out of a bend at a point P with a speed of 25 m/s. The car then travels in a straight line and when $t = 4$ it passes a point Q with a speed of 85 m/s. For the motion from P to Q, its speed, v m/s, can be modelled by the formula $v = At^{2.5} + 11t + C$, where A and C are constants.

(a) Explain why $C = 25$ and find the value of A. [3]

When $t = 0$ s, $v = 25$ m/s

$25 = A(0) + 11(0) + C$

$\therefore C = 25$

When $t = 4$ s: $v = At^{2.5} + 11t + 25$

$85 = A(4)^{2.5} + 11(4) + 25$

$32A = 16$

$A = 0.5$

Hence, $v = \frac{1}{2}t^{2.5} + 11t + 25$

oint P with a speed
h a speed of 85 m/s.
$v = At^{2.5} + 11t + C$,

[3]

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(b) Find the acceleration of the car as it passes Q. [2]

$$v = \frac{1}{2}t^{2.5} + 11t + 25$$

$$\frac{dv}{dt} = \frac{1}{2}(2.5)t^{1.5} + 11$$

$$= 1.25t^{1.5} + 11$$

When $t = 4s$, $a = 1.25(4)^{1.5} + 11$

$$= \underline{21 \text{ m/s}^2}$$

(c) Find the distance PQ to the nearest metre. [4]

$$\int_{t=0}^{t=4} \frac{1}{2}t^{2.5} + 11t + 25 \, dt$$

$$= \left[\frac{\frac{1}{2}t^{3.5}}{3.5} + \frac{11t^2}{2} + 25t \right]_0^4$$

$$= \left[\frac{1}{7}t^{3.5} + \frac{11}{2}t^2 + 25t \right]_0^4$$

$$= \frac{1}{7}(4)^{3.5} + \frac{11}{2}(4)^2 + 25(4)$$

$$= 206.29$$

$$= \underline{206 \text{ m}}$$

9 (a) Prove the identity $\tan x + \cot x = 2\operatorname{cosec} 2x$. [4]

$$\text{LHS} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x}$$

$$= \frac{2}{2\sin x \cos x}$$

$$= \frac{2}{\sin 2x}$$

$$= 2\operatorname{cosec} 2x$$

$$= \text{RHS} \quad (\text{proved})$$

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(b) Solve the

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(b) Solve the equation $\tan x + \cot x = 3$ for $0 \leqslant x \leqslant 2\pi$. [6]

$$2\operatorname{cosec} 2x = 3$$

$$\frac{2}{\sin 2x} = 3$$

$$\sin 2x = \frac{2}{3}$$

Step 1: Basic angle for $2x = 0.72973$ rad.

Step 2: $2x$ lies in 1st or 2nd quad.

Step 3: given $0 \leqslant x \leqslant 2\pi$,

$\therefore 0 \leqslant 2x \leqslant 4\pi$

Step 4: $2x = 0.72973,\ \pi - 0.72973,\ 2\pi + 0.72973$ or $2\pi + (\pi - 0.72973)$

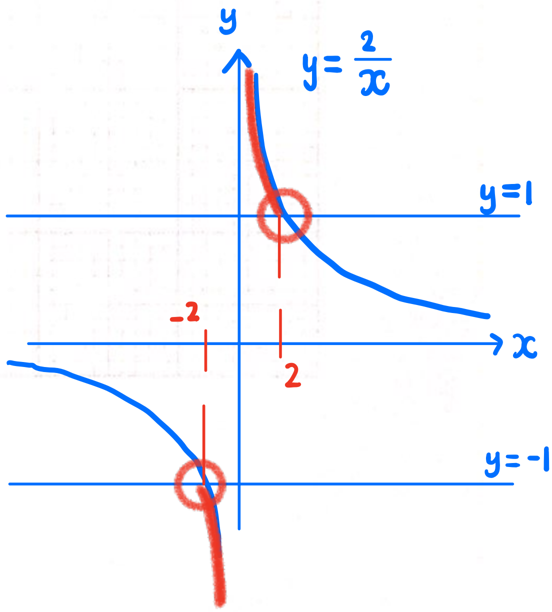
$\therefore$ $\underline{x = 0.365,\ 1.21,\ 3.51 \text{ or } 4.35}$ rad.

(c) Use the result in **part (a)** to find the set of values of the constant k for which there are no solutions to the equation $\tan x + \cot x = k$. [2]

$$2\operatorname{cosec} 2x = k$$

$$\frac{2}{\sin 2x} = k$$

$$\sin 2x = \frac{2}{k}$$



Since $-1 \leqslant \sin 2x \leqslant 1$,

for no solutions, $\frac{2}{k} < -1$ or $\frac{2}{k} > 1$

From graph, $\underline{-2 < k < 0 \text{ or } 0 < k < 2}$

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10 **(a)** When an object falls, it experiences air resistance. For one particular object, the resistance, R, depends upon the velocity, v, of the object and can be modelled by the formula $R = Av^k$, where A and k are constants. The table below shows corresponding values of v and R. It is believed that an error was made in recording one of the values of R.

v	5	10	15	20	25
R	21	55	110	146	200

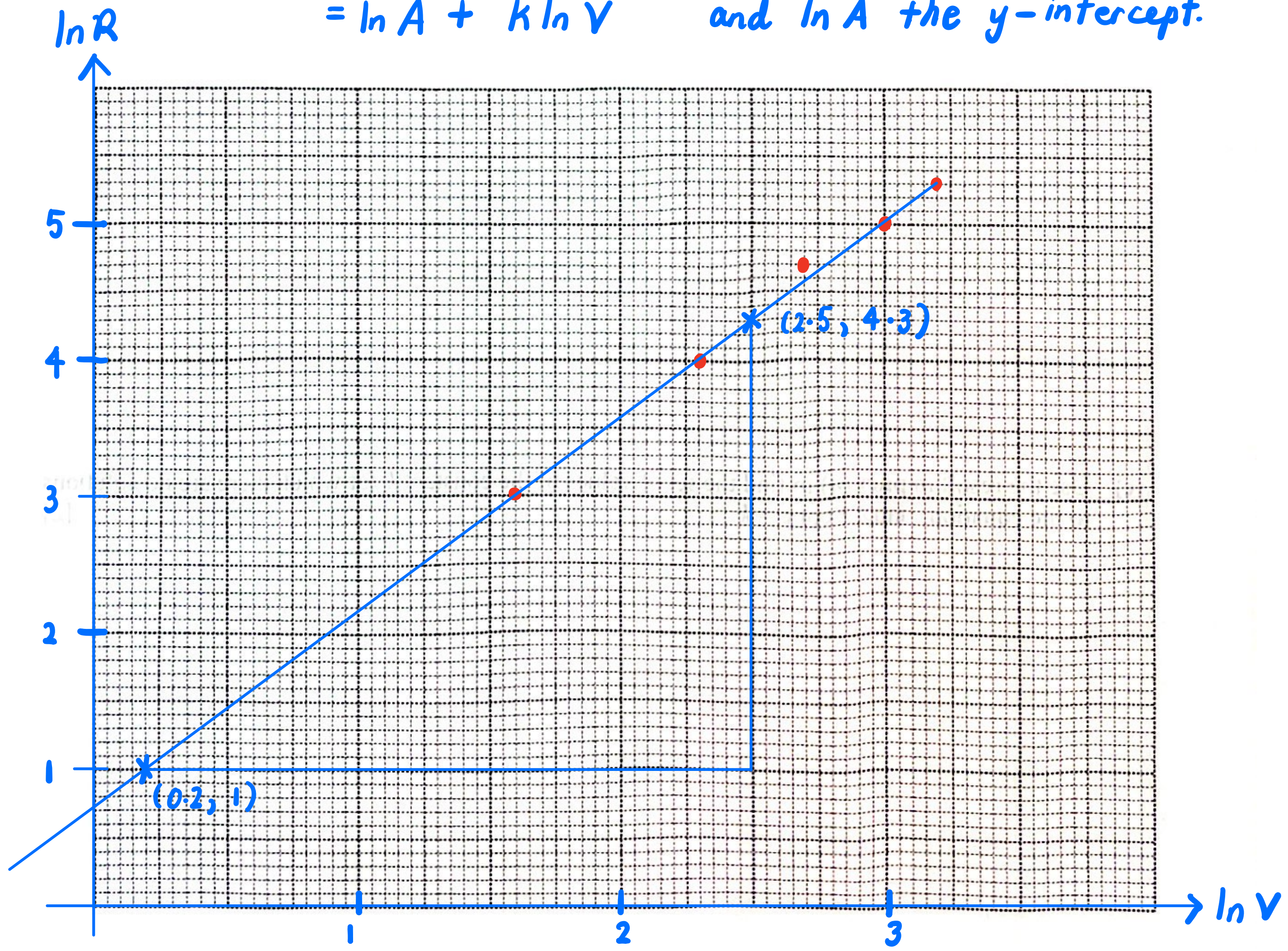
(i) Explain how a straight line graph can be drawn to represent the formula and draw it for the given data. **[4]**

$R = Av^k$

$\ln R = \ln(Av^k)$

$= \ln A + k \ln v$

Plot $\ln R$ against $\ln v$, where k is the gradient, and $\ln A$ the y-intercept.



$\ln V$	1.61	2.30	2.71	3.00	3.22
$\ln R$	3.04	4.01	4.70	4.98	5.30

(ii) From your straight line graph, identify the incorrect reading and suggest a corrected value for R. [2]

From graph, wrong reading of $R = 110$.
corrected reading of $\ln R = 4.6$

$\therefore R = e^{4.6} = 99.5$

(iii) Estimate the value of k. [2]

$k = $ gradient

$= \dfrac{4.3 - 1}{2.5 - 0.2} = 1.43$

(b) Variables x and y are related by the equation $y = ax + \dfrac{b}{x}$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this equation. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]

$y = ax + \dfrac{b}{x}$

$xy = ax^2 + b$

Plot xy (vertical axis) against x^2 (horizontal axis).

The gradient of the straight line $= a$

The y-intercept of the straight line $= b$.

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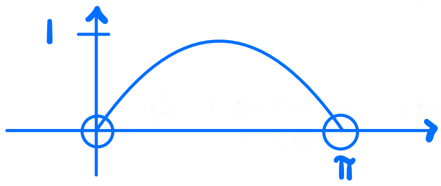
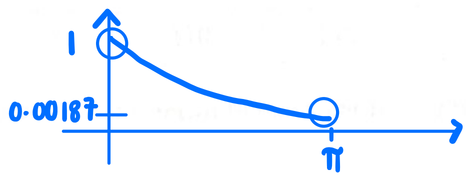
11 The equation of a curve is $y = e^{-2x}\sin x$.

(a) Explain why the curve lies completely above the x-axis for $0 < x < \pi$. [2]

For $0 < x < \pi$, $0 < \sin x < 1$, For $0 < x < \pi$, $1 < e^{-2x} < 0.00187$





Hence $e^{-2x} \times \sin x > 0$ for $0 < x < \pi$.

(b) Find $\frac{dy}{dx}$ and show that the x-coordinate of the stationary point for $0 < x < \pi$ is $\tan^{-1}\frac{1}{2}$. [5]

$$\frac{dy}{dx} = e^{-2x}\cos x + \sin x(-2e^{-2x})$$
$$= e^{-2x}(\cos x - 2\sin x)$$

Let $\frac{dy}{dx} = 0$

$$e^{-2x}(\cos x - 2\sin x) = 0$$

since $e^{-2x} > 0$, $\cos x - 2\sin x = 0$

$$2\sin x = \cos x$$
$$\frac{\sin x}{\cos x} = \frac{1}{2}$$
$$\tan x = \frac{1}{2}$$
$$x = \tan^{-1}\frac{1}{2}$$

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(c) Find $\frac{d^2y}{dx^2}$ and show that $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=0$. [5]

$$\frac{dy}{dx} = e^{-2x}\cos x + \sin x(-2e^{-2x})$$
$$= e^{-2x}(\cos x - 2\sin x)$$

$$\frac{d^2y}{dx^2} = e^{-2x}(-\sin x - 2\cos x) + (\cos x - 2\sin x)(-2)e^{-2x}$$
$$= e^{-2x}(3\sin x - 4\cos x)$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y$$
$$= e^{-2x}(3\sin x - 4\cos x) + 4e^{-2x}(\cos x - 2\sin x) + 5e^{-2x}\sin x$$
$$= (3e^{-2x}\sin x - 8e^{-2x}\sin x + 5e^{-2x}\sin x) - 4e^{-2x}\cos x + 4e^{-2x}\cos x$$
$$= 0.$$

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