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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER	S						INDEX NUMBER					

ADDITIONAL MATHEMATICS

4049/01

Paper 1

October/November 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

[Turn over

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Answer **all** the questions.

1 An item is to be sold online. At the start of the month, it is placed on sale at an initial price of \$300. At the end of each month, if the item hasn't sold, the price is automatically reduced by 5%. It remains on sale at this price for the next month. The item finally sells in the month when the item is first priced below \$120.

How many complete months have elapsed before the item is sold? [5]

Let A be the price of the item.

After 1st month, $A = 0.95(300)$

After 2nd month, $A = 0.95(0.95)(300)$

$= 0.95^2(300)$

After 3rd month, $A = 0.95(0.95^2)(300)$

$= 0.95^3(300)$

After nth month, $A = 0.95^n(300)$

Let $0.95^n(300) < 120$

$0.95^n < \frac{2}{5}$

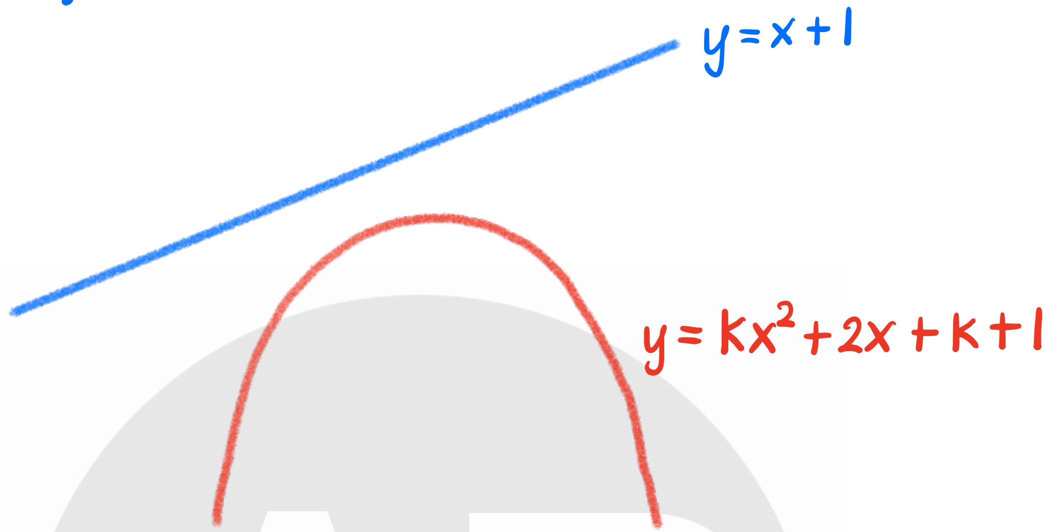
$n \lg 0.95 < \lg \frac{2}{5}$

$n > 17.9$ months

After 18 complete months, the item is sold.

2 Find the set of values of the constant k for which the curve $y = kx^2 + 2x + k + 1$ is completely below the line $y = x + 1$. [6]

Firstly, for a quadratic curve to be completely below the line $y = x + 1$, $k < 0$.



Let $kx^2 + 2x + k + 1 = x + 1$

$kx^2 + x + k = 0$

Since there is no intersection between the line and the curve, $b^2 - 4ac < 0$

$1^2 - 4k(k) < 0$

$4k^2 > 1$

$4k^2 - 1 > 0$

$(2k - 1)(2k + 1) > 0$

$\therefore \; \underline{k < -\frac{1}{2}}$ or $k > \frac{1}{2}$ (rej. because $k < 0$)

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

3 **(a)** **(i)** Factorise $a^3 - b^3$. [1]

(ii) Hence state a factor, other than itself and 1, of $347^3 - 286^3$. [1]

It is given that x and n are positive integers.

(b) Use **part (a)(i)** to prove that $\dfrac{(x+n)^3}{2} - \dfrac{(x-n)^3}{2}$ is divisible by n. [3]

Recap:

Difference of Cubes Formula:

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Sum of Cubes Formula:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(a) (i). $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(ii). $347^3 - 286^3 = (347-286)[347^2 + (347)(286) + 286^2]$

$= (61)(301447)$

Factor $= 61$ or 301447

(b)
$$\frac{(x+n)^3}{2} - \frac{(x-n)^3}{2}$$
$$= \frac{(x+n)^3 - (x-n)^3}{2}$$
$$= \frac{[(x+n)-(x-n)][(x+n)^2 + (x+n)(x-n) + (x-n)^2]}{2}$$
$$= \frac{2n[x^2 + \cancel{2nx} + \cancel{n^2} + x^2 - \cancel{n^2} + x^2 - \cancel{2nx} + n^2]}{2}$$
$$= \frac{\cancel{2}n(3x^2 + n^2)}{\cancel{2}}$$

Since $\dfrac{(x+n)^3}{2} - \dfrac{(x-n)^3}{2} = n(3x^2 + n^2)$,

then n is a factor of $\dfrac{(x+n)^3}{2} - \dfrac{(x-n)^3}{2}$,

hence $\dfrac{(x+n)^3}{2} - \dfrac{(x-n)^3}{2}$ is divisible by n.

4 (a) Differentiate $x^{n+1}\ln x$ with respect to x, where $n \geqslant 0$. [2]

(b) Hence find $\int x^n \ln x dx$. [3]

(a) $$\frac{d}{dx} x^{n+1} \ln x = x^{n+1}\left(\frac{1}{x}\right) + (\ln x)\cdot(n+1)x^n$$
$$= x^n + x^n(\ln x)(n+1)$$

(b) $$\int x^n \ln x \, dx = \frac{1}{n+1}\int x^n(\ln x)(n+1)\, dx$$
$$= \frac{1}{n+1}\int \left[x^n + x^n(\ln x)(n+1)\right] - x^n \, dx$$
$$= \frac{1}{n+1}\left[x^{n+1}\ln x - \frac{x^{n+1}}{n+1}\right] + C$$

5 Two variables x and y are believed to be related by the equation $y = Ax^b$ where A and b are constants. When a set of points (X, Y) are plotted, where X is a function of x and Y is a function of y, the resulting graph is a straight line.

(a) Given that $\ln y$ is plotted on the Y-axis determine, with working, the function of x which must be plotted on the X-axis. [2]

(b) Given that two of the points lying on the straight line are $(1.2, 6.3)$ and $(4.1, 12.1)$ find the values of A and b. [4]

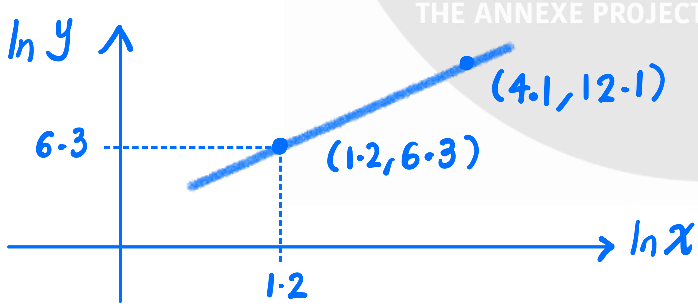
(a) $y = Ax^b$

$$\ln y = \ln Ax^b$$
$$= \ln A + \ln x^b$$
$$= b \ln x + \ln A$$

Plot $\ln y$ against $\underline{\ln x \ (X\text{-axis})}$, with gradient b and y-intercept $\ln A$.

(b) $b = \dfrac{12.1 - 6.3}{4.1 - 1.2}$

$= 2$



The straight line equation is:

$$\ln y = 2 \ln x + \ln A$$

Since $\ln x = 1.2$, and $\ln y = 6.3$

then $6.3 = 2(1.2) + \ln A$

$$\ln A = 6.3 - 2.4$$
$$= 3.9$$
$$A = e^{3.9}$$
$$= 49.402$$
$$= 49.4$$

6 (a) Show that the x-coordinate of the stationary point on the curve $y=\frac{1}{2}x^2-3\sqrt{x}+1$ is $\left(\frac{3}{2}\right)^{\frac{2}{3}}$. [4]

(b) Determine the nature of this stationary point. [3]

(a). $\frac{dy}{dx} = \frac{1}{2}(2)x - 3\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$

$= x - \frac{3}{2}x^{-\frac{1}{2}}$

$= x - \frac{3}{2\sqrt{x}}$

When $\frac{dy}{dx}=0$, $x - \frac{3}{2\sqrt{x}} = 0$

$\frac{2x\sqrt{x}-3}{2\sqrt{x}} = 0$

$\therefore 2x^{\frac{3}{2}} - 3 = 0$

$x^{\frac{3}{2}} = \frac{3}{2}$

$x = \left(\frac{3}{2}\right)^{\frac{2}{3}}$

(b). Using 1st Derivative test,

x	1.31	$\left(\frac{3}{2}\right)^{\frac{2}{3}}$	1.32
$\frac{dy}{dx}$	\	—	/

it is a minimum point.

7 **(a)** Find, in ascending powers of x, the first four terms in the expansion of $\left(\frac{1}{2}-ax\right)^5$, where a is a constant. Simplify the coefficients in your expansion. [4]

(b) In the expansion of $\left(4+\frac{5}{ax}\right)\left(\frac{1}{2}-ax\right)^5$, find the non-zero value of a for which the coefficient of x is twice the coefficient of x^2. [3]

ⓐ $\left(\frac{1}{2}-ax\right)^5 = \left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^4(-ax) + {}^5C_2\left(\frac{1}{2}\right)^3(-ax)^2 + {}^5C_3\left(\frac{1}{2}\right)^2(-ax)^3 + \cdots$

$= \frac{1}{32} - \frac{5}{16}ax + \frac{5}{4}a^2x^2 - \frac{5}{2}a^3x^3 + \cdots$

ⓑ $\left(4+\frac{5}{ax}\right)\left(\frac{1}{2}-ax\right)^5$

gives us coefficient of x^2

$= \left(4+\frac{5}{ax}\right)\left(\frac{1}{32} - \frac{5}{16}ax + \frac{5}{4}a^2x^2 - \frac{5}{2}a^3x^3 + \cdots\right)$

gives us coefficient of x

coefficient of x $= 4\left(-\frac{5}{16}a\right) + \left(\frac{5}{a}\right)\left(\frac{5}{4}a^2\right)$

$= -\frac{5}{4}a + \frac{25}{4}a$

$= 5a$

coefficient of x^2 $= 4\left(\frac{5}{4}a^2\right) + \left(\frac{5}{a}\right)\left(-\frac{5}{2}a^3\right)$

$= 5a^2 - \frac{25}{2}a^2$

$= \frac{-15}{2}a^2$

Given coefficient of x = $2 \times$ coefficient of x^2

$5a = 2\left(\frac{-15}{2}a^2\right)$

$5a = -15a^2$

$a = -3a^2$

$3a^2 + a = 0$

$a(3a+1) = 0$

$a = 0$ (rej.) or $a = -\frac{1}{3}$

8 A curve has equation $\dfrac{(x-a)^2}{2}+(y-1)^2=1$, where a is a constant.

(a) Find the set of values of a for which the line $y=x+1$ intersects this curve at two distinct points. [5]

(b) Deduce the two values of a for which the line $y=x+1$ is a tangent to the curve. Show your working clearly. [2]

(a) Step 1 : Combined both equations to the form $ax^2+bx+c=0$

$$\frac{(x-a)^2}{2}+(y-1)^2=1 \quad \text{— (1)}$$

$$y=x+1 \quad \text{— (2)}$$

Sub (2) into (1) : $\dfrac{(x-a)^2}{2}+(x+1-1)^2=1$

$$(x-a)^2+2x^2=2$$

$$x^2-2ax+a^2+2x^2-2=0$$

$$3x^2-2ax+(a^2-2)=0$$

Step 2 : $b^2-4ac>0$

$$(-2a)^2-4(3)(a^2-2)>0$$

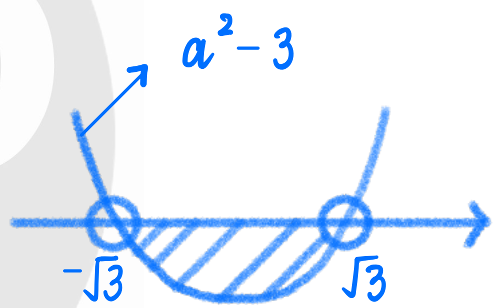
$$4a^2-12a^2+24>0$$

$$8a^2<24$$

$$a^2<3$$

$$a^2-3<0$$

$$(a-\sqrt{3})(a+\sqrt{3})<0$$



$$\therefore\ -\sqrt{3}<a<\sqrt{3}$$

(b) $b^2-4ac=0$

$$(-2a)^2-4(3)(a^2-2)=0$$

$$4a^2-12a^2+24=0$$

$$8a^2=24$$

$$a^2=3$$

$$a=\pm 3$$

9 (a) Prove that $\dfrac{1+\sin 2\theta}{\sin\theta} - \dfrac{1+\cos 2\theta}{\cos\theta} = \operatorname{cosec}\theta$. [4]

(b) Hence solve $\dfrac{1+\sin 2\theta}{\sin\theta} - \dfrac{1+\cos 2\theta}{\cos\theta} = 2\sin\theta$ for $\pi < \theta < \frac{3}{2}\pi$. [3]

(a)
$$\text{LHS} = \frac{1+\sin 2\theta}{\sin\theta} - \frac{1+\cos 2\theta}{\cos\theta}$$
$$= \frac{(1+\sin 2\theta)\cos\theta - (1+\cos 2\theta)\sin\theta}{\sin\theta\cos\theta}$$
$$= \frac{(1+2\sin\theta\cos\theta)\cos\theta - (\cancel{1}+2\cos^2\theta-\cancel{1})\sin\theta}{\sin\theta\cos\theta}$$
$$= \frac{\cos\theta + 2\sin\theta\cancel{\cos^2\theta} - 2\sin\theta\cancel{\cos^2\theta}}{\sin\theta\cos\theta}$$
$$= \frac{\cancel{\cos\theta}}{\sin\theta\cancel{\cos\theta}}$$
$$= \frac{1}{\sin\theta}$$
$$= \operatorname{cosec}\theta$$
$$= \text{RHS (shown)}$$

(b) Hence, $\operatorname{cosec}\theta = 2\sin\theta$
$$\frac{1}{\sin\theta} = 2\sin\theta$$
$$2\sin^2\theta = 1$$
$$\sin^2\theta = \frac{1}{2}$$
$$\sin\theta = \pm\sqrt{\frac{1}{2}}$$
$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ or } \frac{7\pi}{4}$$

Since $\pi < \theta < \frac{3}{2}\pi$, $\theta = \frac{5\pi}{4}$

10 **(a)** Find, in terms of the positive constant a, the equation of the normal to the curve $y = 2\sin ax - 3\cos 2x$ at the point P where $x = 0$. [7]

(b) The normal cuts the x-axis at Q. The area of triangle OPQ, where O is the origin, is 27 units^2. Find the value of a. [3]

(a) Step 1: Find coordinate of P

When $x = 0$, $y = 0 - 3\cos 0$
$= -3$

$\therefore P = (0, -3)$

Step 2: Find $\frac{dy}{dx}$ at P

$y = 2\sin ax - 3\cos 2x$

$\frac{dy}{dx} = 2a\cos ax + 6\sin 2x$

at P, $\frac{dy}{dx} = 2a(1) + 6(0)$
$= 2a$

Step 3:

Hence, gradient of normal at P $= -\frac{1}{2a}$

Step 4: Equation of normal at P

$y - (-3) = \frac{-1}{2a}(x - 0)$

$y = \frac{-1}{2a}x - 3$

(b) Let $y = 0$, $\frac{-1}{2a}x - 3 = 0$

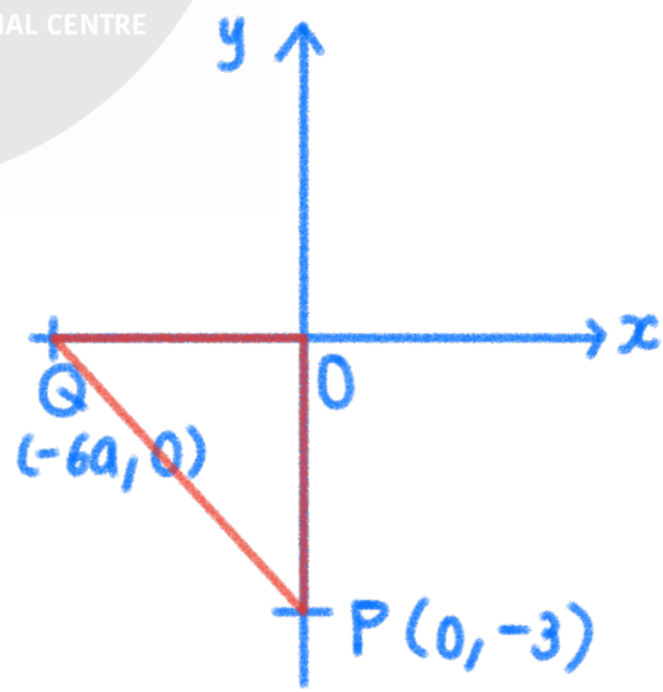
$\frac{x}{2a} = -3$

$x = -6a$

$\therefore Q = (-6a, 0)$

Area of $\triangle OPQ = \frac{1}{2} \times 6a \times 3$
$= 9a$

$\therefore 9a = 27$
$a = 3$



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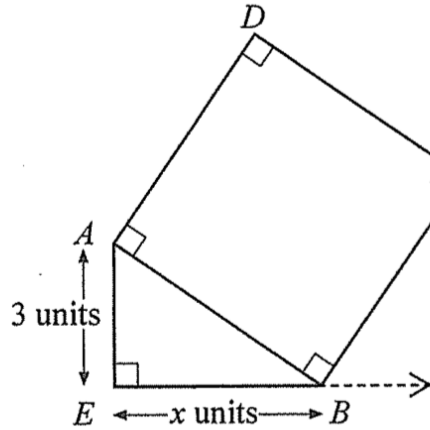


A square $ABCD$ is part of a computer animation. The side AB is the hypotenuse of a right-angled triangle AEB with angle $AEB = 90°$. The points A and E are fixed and do not change during the animation.

The length of AE is always 3 units and angle AEB is always $90°$.

During the animation B moves directly away from E along the line perpendicular to AE and, at time t seconds, the distance EB is x units.

It is given that x is increasing at a constant rate of 2 units/s.

(a) Find the rate of change of the length AB, in units/s, when $x = 4$. [5]

(b) The rate of change of the area of the square cannot exceed 80 units^2/s. Find the maximum possible value for x. [3]

(a) $AB = \sqrt{3^2 + x^2}$

$= \sqrt{9 + x^2}$

$\dfrac{dAB}{dx} = \frac{1}{2}(9 + x^2)^{-\frac{1}{2}}(2x) = \dfrac{x}{\sqrt{9 + x^2}}$

Given: $\dfrac{dx}{dt} = 2$ units/s

Find: $\dfrac{dAB}{dt}$ when $x = 4$

Chain-rule: $\dfrac{dAB}{dt} = \dfrac{dAB}{dx} \cdot \dfrac{dx}{dt}$

$= \dfrac{4}{\sqrt{9+16}} \cdot 2$

$= \dfrac{8}{5}$ units/s

(b) Area of square $= \sqrt{9+x^2} \cdot \sqrt{9+x^2}$
$= (9+x^2)$ units^2.

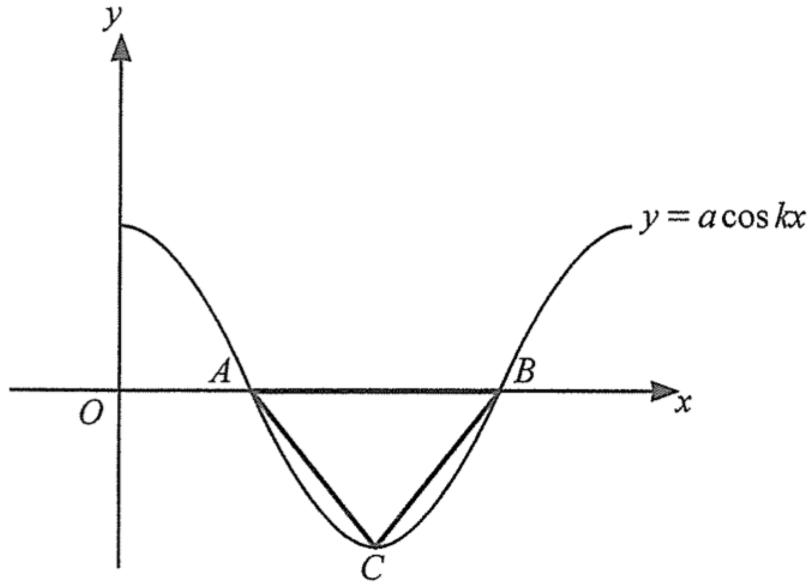
$$\frac{dA}{dx} = 2x$$

$$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}$$
$$= 2x \cdot 2$$
$$= 4x \text{ units}^2/s$$

Given $\frac{dA}{dt} \leqslant 80$

$$4x \leqslant 80$$
$$x \leqslant 20 \text{ units}$$

$\therefore$ max. value of $x = 20$ units.



The diagram shows part of the curve $y = a\cos kx$ where a and k are positive constants and x is in radians. The curve crosses the positive x-axis at A and B.

(a) **(i)** Find, in terms of k, the x-coordinates of A and of B. [2]

Point C is the minimum point on the curve between A and B.
Triangle ABC is equilateral.

(ii) Prove that $a = \dfrac{\pi\sqrt{3}}{2k}$. [3]

(b) Given that the area of triangle ABC is 1 unit^2, show that $k = \dfrac{(3)^{\frac{1}{4}}\pi}{2}$. Hence find the amplitude and period of the curve. [3]

ⓐ (i). When $y = 0$, $a\cos kx = 0$

$$\cos kx = 0$$

$$kx = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$\therefore x = \frac{\pi}{2k} \text{ or } \frac{3\pi}{2k}$$

Hence, x – coordinate of $A = \frac{\pi}{2k}$ and

x – coordinate of $B = \frac{3\pi}{2k}$.

(ii). by midpoint theorem,

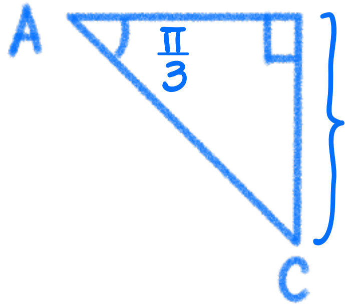
x – coordinate of $C = \frac{\pi}{k}$

Length of $AB = \frac{3\pi}{2k} - \frac{\pi}{2k} = \frac{\pi}{k}$

$$\sin\frac{\pi}{3} = \frac{a}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{a}{AC}$$

$$AC = \frac{2a}{\sqrt{3}}$$



a (as amplitude of curve is a)

Length of AB = Length of AC

$$\frac{\pi}{k} = \frac{2a}{\sqrt{3}}$$

$$2ak = \pi\sqrt{3}$$

$$a = \frac{\pi\sqrt{3}}{2k} \quad \text{(shown).}$$

(b) Area of $\triangle ABC = \frac{1}{2} \times AB \times a$

$$= \frac{1}{2} \times \frac{\pi}{k} \times a$$

$$= \frac{\pi a}{2k}$$

<u>Given area of $\triangle ABC = 1$ units^2, and $a = \frac{\pi\sqrt{3}}{2k}$</u>

$$1 = \frac{\pi}{2k}\left(\frac{\pi\sqrt{3}}{2k}\right)$$

$$4k^2 = \pi^2\sqrt{3}$$

$$k^2 = \frac{\pi^2\sqrt{3}}{4}$$

$$k = \pm\frac{\pi}{2}(3)^{\frac{1}{4}}$$

Since k is positive, $k = \frac{(3)^{\frac{1}{4}}\pi}{2}$

- amplitude of curve $= a = \frac{\pi\sqrt{3}}{2\left[\frac{(3)^{\frac{1}{4}}\pi}{2}\right]} = \frac{\cancel{\pi}(3^{\frac{1}{2}})}{\cancel{\pi}(3^{\frac{1}{4}})} = 3^{\frac{1}{2}-\frac{1}{4}} = 3^{\frac{1}{4}}$

- period of curve $= \frac{2\pi}{k} = \frac{2\pi}{\left[\frac{(3)^{\frac{1}{4}}\pi}{2}\right]} = \frac{4\cancel{\pi}}{(3^{\frac{1}{4}})\cancel{\pi}} = \frac{4}{3^{\frac{1}{4}}}$

13 Points $A(-1,7)$ and B lie on a circle.
The chord AB of this circle has equation $2y-3x=17$.
The centre of the circle lies on the line $y=-2$.
The diameter of the circle parallel to AB passes through the point $\left(-\frac{1}{2}, \frac{1}{4}\right)$.

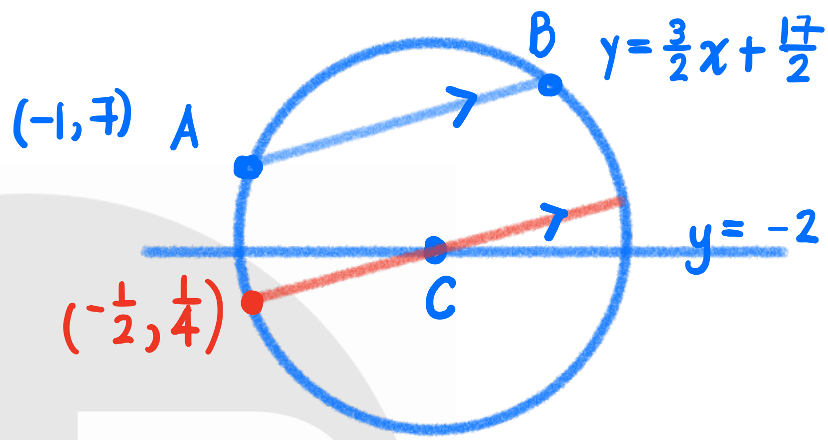
(a) Find the equation of the circle. [7]

(b) The point $(7, \alpha)$ lies **inside** the circle. Find the set of possible values for α. [2]

(a) $2y - 3x = 17$

$2y = 3x + 17$

$y = \frac{3}{2}x + \frac{17}{2}$



Equation of red line:

$y - \frac{1}{4} = \frac{3}{2}\left(x + \frac{1}{2}\right)$

$y = \frac{3}{2}x + 1$ —— (1)

$y = -2$ —— (2)

$\therefore \frac{3}{2}x + 1 = -2$

$\frac{3}{2}x = -3$

$x = -2$

Hence, centre of circle $= (-2, -2)$

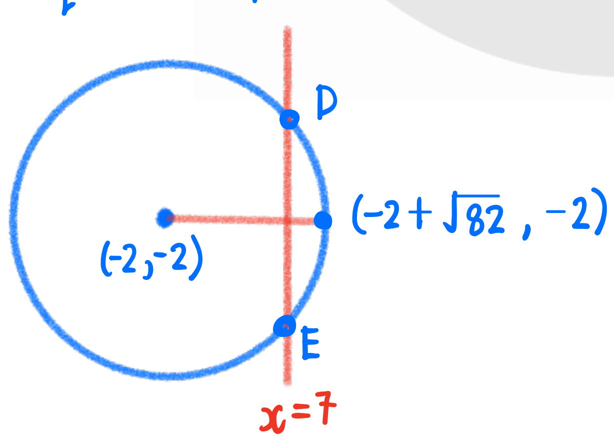
radius of circle = length of AC $= \sqrt{(-1+2)^2 + (7+2)^2}$

$= \sqrt{1 + 81}$

$= \sqrt{82}$

$\therefore$ equation of circle: $(x+2)^2 + (y+2)^2 = 82$.

(b)



when $x = 7$,

$(7+2)^2 + (y+2)^2 = 82$

$(y+2)^2 = 82 - 81$

$y + 2 = \pm 1$

$y = -3$ or -1

$\therefore D = (7,-1)$, $E = (7,-3)$

If $(7, \alpha)$ lies inside the circle, $-3 < \alpha < -1$