MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/02

Paper 2

October/November 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

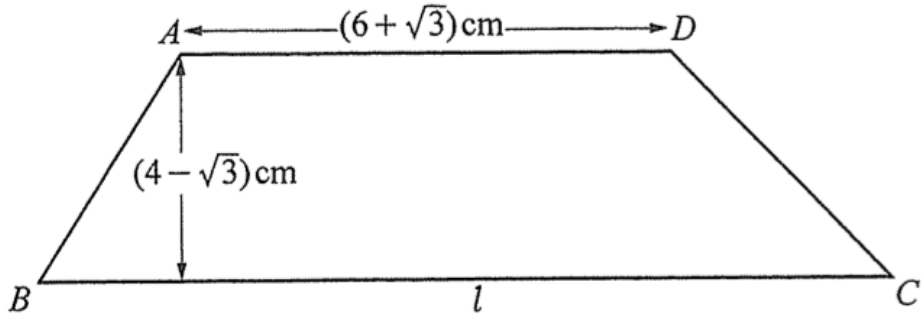
The total number of marks for this paper is 90.

Printed and published by Singapore Asia Publishers Pte Ltd
for the Singapore Examinations and Assessment Board under licence from the Chancellor, Masters and Scholars of the University of Cambridge (acting through the University of Cambridge International Examinations Syndicate) either solely or jointly with the Government of Singapore (acting through the Ministry of Education, Singapore).

(2025)5
OL Additional Mathematics

Answer **all** the questions.

1 **Do not use a calculator in this question.**



The diagram shows a trapezium $ABCD$ with BC and AD parallel. The length of AD is $(6+\sqrt{3})$ cm and the perpendicular height of the trapezium is $(4-\sqrt{3})$ cm. The length of BC is l cm. Given that the trapezium has area 26 cm^2, find the exact length of BC. Give your answer in the form $a+b\sqrt{3}$ where a and b are integers. [6]

Area of trapezium $= \frac{h}{2}$ (sum of parallel lengths)

$$26 = \tfrac{1}{2} \times (4-\sqrt{3}) \times [(6+\sqrt{3}) + l]$$

$$52 = (4-\sqrt{3})(6+\sqrt{3}) + (4-\sqrt{3})l$$

$$52 = 24 - 2\sqrt{3} - 3 + (4-\sqrt{3})l$$

$$(31+2\sqrt{3}) = (4-\sqrt{3})l$$

$$\therefore \quad l = \frac{(31+2\sqrt{3})}{(4-\sqrt{3})} \times \frac{(4+\sqrt{3})}{(4+\sqrt{3})}$$

$$= \frac{124 + 31\sqrt{3} + 8\sqrt{3} + 6}{4^2 - 3}$$

$$= \frac{130 + 39\sqrt{3}}{13}$$

$$= (10 + 3\sqrt{3}) \text{ cm}$$

2 On a winter's day the temperature, T °C, h hours after measurement began, is modelled by the formula
$T = 4 + 2h - \frac{1}{2}h^2$, where $0 \leqslant h \leqslant 8$.

(a) By expressing T in the form $A - B(h+C)^2$, where A, B and C are constants, find the time at which the temperature is greatest and state the value of this temperature. [5]

(b) On the following day, and at the same times as before, the temperatures are found to be half their previous day's values. State the time at which the temperature is greatest and the value of this temperature. [2]

(a)
$$\begin{aligned} T &= -\tfrac{1}{2}h^2 + 2h + 4 \\ &= -\tfrac{1}{2}(h^2 - 4h) + 4 \\ &= -\tfrac{1}{2}[(h-2)^2 - 2^2] + 4 \\ &= -\tfrac{1}{2}(h-2)^2 + 2 + 4 \\ &= 6 - \tfrac{1}{2}(h-2)^2 \end{aligned}$$
, where $A = 6$, $B = \frac{1}{2}$, $C = -2$

T is greatest at $h = 2$.
$T_{max} = 6$ °C

(b)
$$\begin{aligned} T &= \tfrac{1}{2}\left[6 - \tfrac{1}{2}(h-2)^2\right] \\ &= 3 - \tfrac{1}{4}(h-2)^2 \end{aligned}$$

T is greatest at $h = 2$.
$T_{max} = 3$ °C

3 The gradient of a curve, at a point (x, y) on the curve, is given by $e^x(3e^{2x} - e^{-4x}) - \cos 2x$. The curve passes through the origin. Find y in terms of x. [7]

Step 1: Find the equation of the curve

$$\text{Given } \frac{dy}{dx} = e^x(3e^{2x} - e^{-4x}) - \cos 2x$$
$$= 3e^{3x} - e^{-3x} - \cos 2x$$

$$y = \int 3e^{3x} - e^{-3x} - \cos 2x \, dx$$
$$= \frac{3e^{3x}}{3} - \frac{e^{-3x}}{-3} - \frac{\sin 2x}{2} + C$$
$$= e^{3x} + \frac{1}{3}e^{-3x} - \frac{1}{2}\sin 2x + C$$

Step 2 : Solve C
since (0,0) lies on the curve,

$$0 = e^0 + \frac{1}{3}e^0 - \frac{1}{2}\sin 0 + c$$
$$0 = 1 + \frac{1}{3} + c$$
$$c = -\frac{4}{3}$$

$$\therefore \; y = e^{3x} + \frac{1}{3}e^{-3x} - \frac{1}{2}\sin 2x - \frac{4}{3}$$

4 Do not use a calculator in this question.

(a) It is given that $f(x)=x^3+px^2+qx-6$, where p and q are constants.
$(x+1)$ is a factor of $f(x)$.
When $f(x)$ is divided by $(x+2)$ there is a remainder of 4.

Find the values of p and q. [6]

(b) It is given that $3x-1$ is a factor of $g(x)=9x^3-18x^2-x+2$. Express $g(x)$ as a product of three linear factors. [3]

(a) Given $f(x) = x^3 + px^2 + qx - 6$

$$f(-1) = (-1)^3 + p(-1)^2 + q(-1) - 6 = 0$$
$$-1 + p - q - 6 = 0$$
$$p - q = 7 \quad \text{——} \; (1)$$

$$f(-2) = (-2)^3 + p(-2)^2 + q(-2) - 6 = 4$$
$$-8 + 4p - 2q - 6 = 4$$
$$4p - 2q = 18$$
$$2p - q = 9 \quad \text{——} \; (2)$$

(2) − (1): $p = 2$

Sub $p = 2$ into (1): $2 - q = 7$

$q = -5$

(b)

$$\begin{array}{r|l} & 3x^2 - 5x - 2 \\ \hline 3x-1 & 9x^3 - 18x^2 - x + 2 \\ & -(9x^3 - 3x^2) \\ \hline & -15x^2 - x + 2 \\ & -(-15x^2 + 5x) \\ \hline & -6x + 2 \\ & -(-6x + 2) \\ \hline & 0 \end{array}$$

$3x$	1	x
		$+$
x	-2	$-6x$
$3x^2$	-2	$-5x$

$$g(x) = 9x^3 - 18x^2 - x + 2$$
$$= (3x-1)(3x^2 - 5x - 2)$$
$$= (3x-1)(3x+1)(x-2)$$

5 Given that $y=\dfrac{1-e^{-x}}{1+e^{-x}}$, prove that y increases for all real values of x. [6]

$$\frac{dy}{dx} = \frac{(1+e^{-x})(-e^{-x})(-1) - (1-e^{-x})(e^{-x})(-1)}{(1+e^{-x})^2}$$
$$= \frac{e^{-x}(1+e^{-x}) + e^{-x}(1-e^{-x})}{(1+e^{-x})^2}$$
$$= \frac{e^{-x}(1+\cancel{e^{-x}}+1-\cancel{e^{-x}})}{(1+e^{-x})^2}$$
$$= \frac{2}{e^{x}(1+e^{-x})^2}$$

Since $e^x > 0$, and $(1+e^{-x})^2 > 0$ for all real values of x, then $\frac{dy}{dx} > 0$ for all real values of x.
Hence, y increases for all real values of x.

6 Solve the equation $2\operatorname{cosec}^2\theta - \cot\theta - 3 = 0$ for $0° \leqslant \theta \leqslant 360°$. [7]

Trigo. Identity
$1+\cot^2\theta = \operatorname{cosec}^2\theta$

$$2[1+\cot^2\theta] - \cot\theta - 3 = 0$$
$$2\cot^2\theta - \cot\theta - 1 = 0$$
$$(2\cot\theta + 1)(\cot\theta - 1) = 0$$
$$\cot\theta = -\tfrac{1}{2} \quad \text{or} \quad \cot\theta = 1$$
$$\tan\theta = -2 \quad \text{or} \quad \tan\theta = 1$$

For $\tan\theta = -2$:
θ lies in 2nd or 4th quad,
basic angle $\alpha = \tan^{-1} 2$
$= 63.435°$
$\theta = 180° - 63.435°$ or $\theta = 360° - 63.435°$
$= 116.6°$ or $296.6°$

For $\tan\theta = 1$:
basic angle $\alpha = \tan^{-1} 1 = 45°$
θ lies in 1st or 3rd quad, $\theta = 45°$ or $180° + 45° = 225°$

7 A particle is travelling in a straight line. Its acceleration, a m/s^2, at time t seconds is given by

$$a = (2t+1)^{-1} + 0.3t + 0.2 \text{ for } 0 \leqslant t \leqslant 10.$$

At $t = 0$ the velocity of the particle is -3 m/s.
Find the velocity of the particle when $t = 5$. [7]

$a = (2t+1)^{-1} + 0.3t + 0.2$

$v = \int (2t+1)^{-1} + 0.3t + 0.2 \, dt$

$= \frac{\ln(2t+1)}{2} + \frac{0.3t^2}{2} + 0.2t + C$

$= \frac{1}{2}\ln(2t+1) + 0.15t^2 + 0.2t + C$

Given $v = -3$ m/s when $t = 0$,

$-3 = \frac{1}{2}\ln 1 + C$

$C = -3$

$\therefore v = \frac{1}{2}\ln(2t+1) + 0.15t^2 + 0.2t - 3$

When $t = 5$, $v = \frac{1}{2}\ln 11 + 0.15(25) + 0.2(5) - 3$

$= 2.95$ m/s

8 (a) Solve the equation $2^p = 5(3^{2-p})$. [4]

(b) Solve the simultaneous equations

$$9(2^x)+2(3^{y+1}) = 38,$$
$$15(2^{x-1})-21(3^y) = 23.$$ [5]

(a)
$$2^p = 5\left(\frac{3^2}{3^p}\right)$$
$$2^p \cdot 3^p = 5 \times 9$$
$$(2\times 3)^p = 45$$
$$6^p = 45$$
$$\lg 6^p = \lg 45$$
$$p \lg 6 = \lg 45$$
$$\therefore \quad p = \frac{\lg 45}{\lg 6} = 2.12$$

(b) Let $u = 2^x$ and $v = 3^y$:

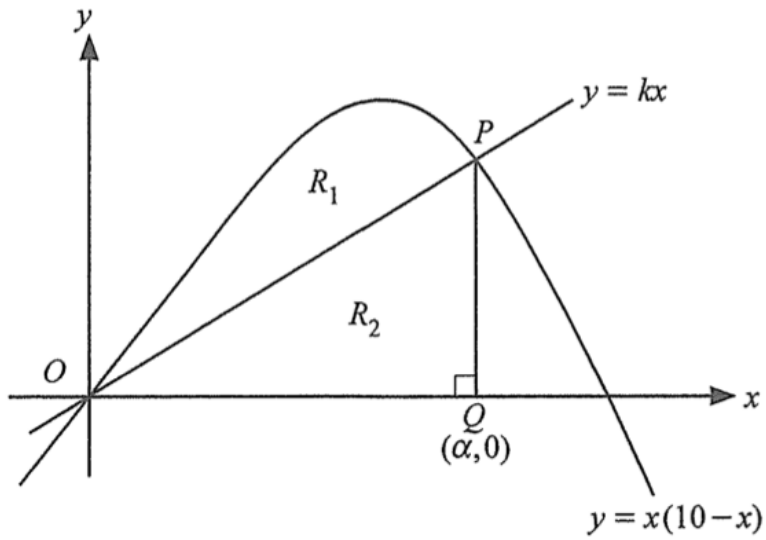
$$9(2^x) + 2(3^{y+1}) = 38$$
$$9u + 2(3^y \times 3) = 38$$
$$9u + 6v = 38 \quad \text{—— } (1)$$

$$15(2^{x-1}) - 21(3^y) = 23$$
$$15\left(\frac{2^x}{2}\right) - 21v = 23$$
$$\frac{15}{2}u - 21v = 23$$
$$15u - 42v = 46 \quad \text{—— } (2)$$
$$(1)\times 7: \quad 63u + 42v = 266 \quad \text{—— } (3)$$

$$(2)+(3): \quad 78u = 312$$
$$u = 4$$
$$2^x = 4$$
$$\therefore \quad x = 2$$

$$(1): \quad 9(4) + 6v = 38$$
$$6v = 38 - 36$$
$$v = \frac{1}{3}$$
$$3^y = 3^{-1}$$
$$\therefore \quad y = -1$$

9



The diagram shows the line $y = kx$, where k is a constant, $k < 10$, and the curve $y = x(10-x)$. The line intersects the curve at the origin O and at point P. Point $Q(\alpha, 0)$, where α is a constant, lies on the x-axis such that PQ is parallel to the y-axis.

R_1 is the area of the region enclosed by the curve and the line OP.

(a) By finding k in terms of α, show that $R_1 = \dfrac{\alpha^3}{6}$. [7]

R_2 is the area of the region enclosed by the lines OP, PQ and the x-axis.

(b) Find the numerical value of k if $R_1 = R_2$. [3]

(a) Step 1 : Find intersection point P

Sub $y = kx$ into $y = x(10-x)$

$\therefore \quad kx = x(10-x)$

$kx - 10x + x^2 = 0$

$x(x + k - 10) = 0$

$x = 0$ (rej.) or $x = 10 - k$

Since P and Q share the same x-coordinate,

$10 - k = \alpha$

$k = 10 - \alpha$

When $x = 10 - k$, $y = k(10-k)$ $\quad \therefore P = (10-k, k(10-k))$

Step 2: Find area R_1

$$\text{area } R_1 = \int_0^{\alpha} x(10-x)\,dx - \text{area } R_2$$

$$= \int_0^{\alpha} 10x - x^2\,dx - \left[\frac{1}{2} \times \alpha \times k(10-k)\right]$$

$$= \left[\frac{10x^2}{2} - \frac{x^3}{3}\right]_0^{\alpha} - \frac{k\alpha}{2}(10-k)$$

$$= \left[5\alpha^2 - \frac{1}{3}\alpha^3\right] - \frac{k\alpha}{2}(10-k)$$

$$= 5\alpha^2 - \frac{1}{3}\alpha^3 - \frac{(10-\alpha)\alpha}{2} \cdot \alpha$$

$$= 5\alpha^2 - \frac{1}{3}\alpha^3 - \frac{10\alpha^2 - \alpha^3}{2}$$

$$= \cancel{5\alpha^2} - \frac{1}{3}\alpha^3 - \cancel{5\alpha^2} + \frac{1}{2}\alpha^3$$

$$= \frac{1}{6}\alpha^3 \quad \text{(shown).}$$

(b) $R_1 = R_2$

$$\therefore \frac{1}{6}\alpha^3 = \frac{k\alpha}{2}(10-k)$$

$$\frac{1}{6}\alpha^3 = \frac{(10-\alpha)\alpha}{2} \cdot \alpha$$

$$\frac{1}{6}\alpha^3 = 5\alpha^2 - \frac{1}{2}\alpha^3$$

$$\frac{2}{3}\alpha^3 - 5\alpha^2 = 0$$

$$2\alpha^3 - 15\alpha^2 = 0$$

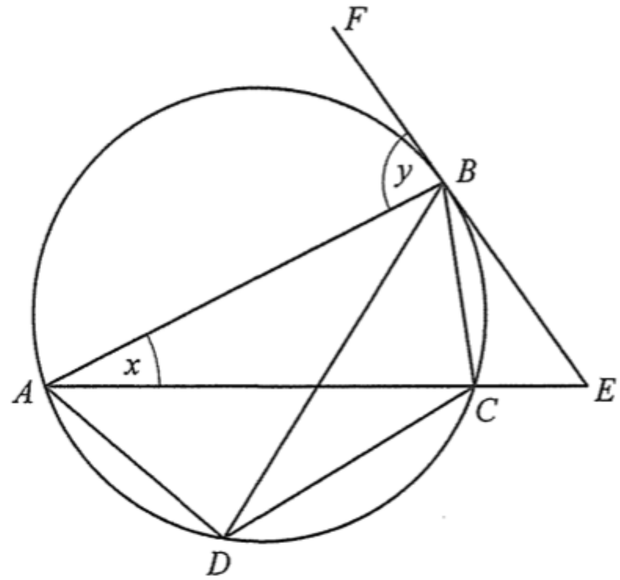
$$\alpha^2(2\alpha - 15) = 0$$

$\alpha = 0$ (rej.) or $\alpha = 7.5$

Since $k = 10 - \alpha$

$k = 2.5$

THE ANNEXE PROJECT
EDUCATIONAL CENTRE
ESTD 2008



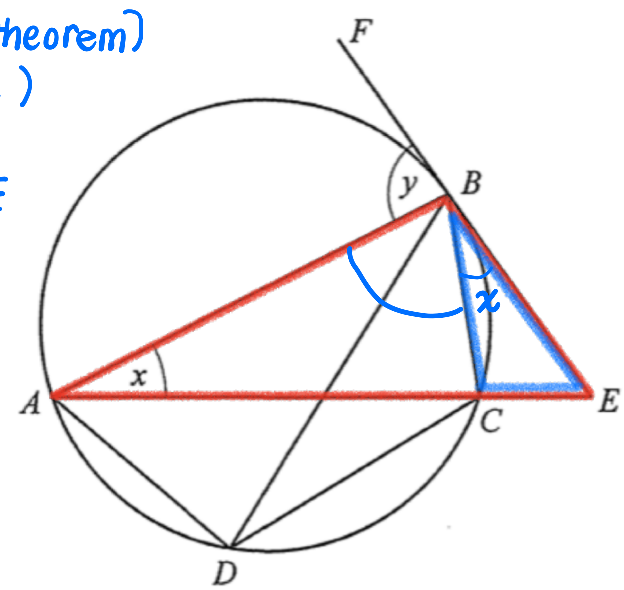
The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of a circle. The tangent to the circle at B meets the line AC extended at the point E. Angle $BAC = x$ and angle $FBA = y$, where F is a point on EB extended.

(a) Prove that triangle ABE is similar to triangle BCE. [3]

(b) **(i)** Prove that angle $ADC = x + y$. [3]

(ii) If DC is parallel to AB prove that $AD = BC$. [4]

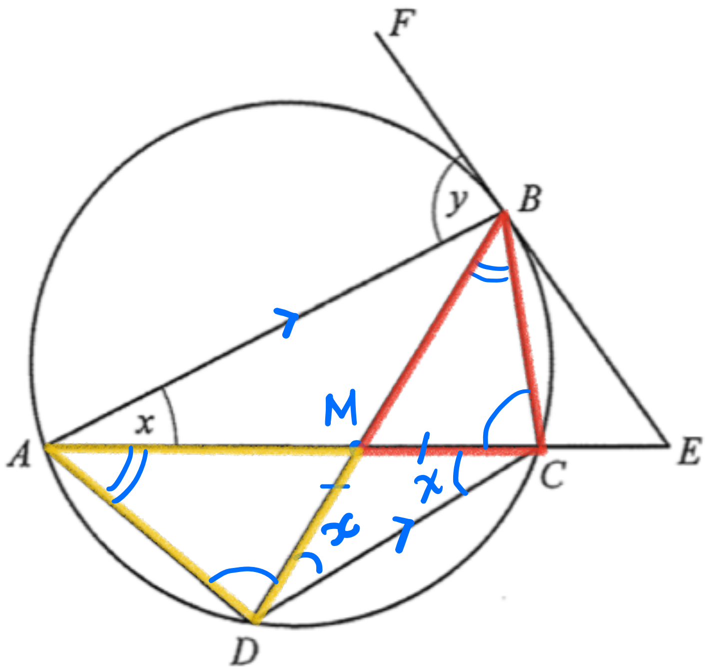
(a) $\angle CBE = \angle BAE = x$ (alt. seg. theorem)
$\angle BEC = \angle AEB$ (common $\angle$)
By A-A test,
$\triangle ABE$ is similar to $\triangle BCE$



(b) i. $\angle ABC = 180° - y - \angle CBE$
$= 180° - y - x$
($\angle$s on a str. line)

$\angle ADC = 180° - \angle ABC$
$= 180° - (180° - y - x)$
$= x + y$
($\angle$s in opp. segments)

ii.



Let the intersection between AC and BD be M.

Step 1: To prove $\triangle ADM$ is congruent to $\triangle BCM$.

$\angle BAC = \angle BDC = x$ ($\angle$s in the same segment)

$\angle BAC = \angle ACD = x$ (AB // DC, alt. $\angle$s)

$\therefore$ $\triangle MDC$ is isos. $\triangle$ $\Rightarrow$ $DM = CM$ (side)

$\angle DAM = \angle CBM$ ($\angle$s in same segment) (angle)

$\angle ADM = \angle BCM$ ($\angle$s in same segment) (angle)

By AAS congruent test,

$\triangle ADM$ is congruent to $\triangle BCM$.

Hence, $AD = BC$

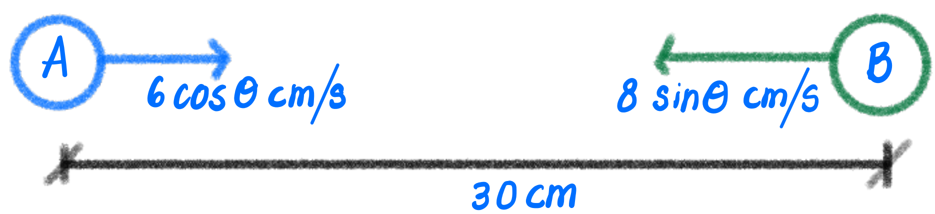
11 At time $t = 0$ seconds, two objects are at points A and B which are 30 cm apart. The objects are moving directly towards one another in a straight line. Their constant speeds are $6\cos\theta$ cm/s and $8\sin\theta$ cm/s respectively, for a certain value of θ measured in degrees. The radii of the objects may be ignored.

(a) Explain why θ must be acute. [1]

(b) In the case when the objects collide at the midpoint of AB, find the time taken for the objects to collide. [3]

(c) **(i)** If, instead, the objects collide when $t = 4$ seconds show that $3\cos\theta + 4\sin\theta = 3.75$. [2]

(ii) By writing $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute, find the numerical values of the speeds of the two objects. [6]



(a) If θ is obtuse, speed of A will be negative and A would be moving left, in the same direction as B (as shown above).

(b) Let t (in s) be the time the objects collide

$(6\cos\theta)t = 15$ —— (1)

$(8\sin\theta)t = 15$ —— (2)

$\frac{(2)}{(1)}$: $\frac{4}{3}\tan\theta = 1$

$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\frac{3}{4} = 36.870$

(1) : $(6\cos 36.870)t = 15$

$t = 3.125$ s

(c) i). $(6\cos\theta)4 + (8\sin\theta)(4) = 30$

$24\cos\theta + 32\sin\theta = 30$

$3\cos\theta + 4\sin\theta = 3.75$ (shown)

ii). $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$

$R = \sqrt{3^2 + 4^2} = 5$

$\alpha = \tan^{-1}\frac{4}{3} = 53.130$

$\therefore 3\cos\theta + 4\sin\theta = 5\cos(\theta - 53.130°)$

Hence, $5\cos(\theta - 53.130^\circ) = 3.75$

$\theta - 53.130^\circ = \cos^{-1}\frac{3}{4}$

$\theta - 53.130^\circ = 41.410^\circ$

Since $\cos(\theta - 53.130^\circ) > 0$,

$\theta - 53.130^\circ$ lies in 1st or 4th quad,

$\theta - 53.130^\circ = 41.410^\circ$ or $\theta - 53.130^\circ = 360^\circ - 41.410^\circ$

$\theta = 94.540^\circ$ (rej.) or $\theta = 371.72^\circ$ (out of range)

$\therefore \theta = 371.72^\circ - 360^\circ$

$= 11.72^\circ$

$\therefore$ speed of $A = 6\cos 11.72^\circ$

$= 5.87$ cm/s

speed of $B = 8\sin 11.72^\circ$

$= 1.63$ cm/s

THE ANNEXE PROJECT
EDUCATIONAL CENTRE
ESTD 2008