

MATHEMATICS

Paper 1

9758/01

October/November 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

1 A sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_1 = a,$$

$$u_{r+1} = 3u_r + 5, \quad r \geq 1,$$

where a is a constant.

(a) Find expressions for u_2 and u_3 in terms of a .

[2]

(b) It is given that $\sum_{r=1}^4 u_r = 110$. Find the value of a .

[3]

$$\begin{aligned} \textcircled{a} \quad u_2 &= 3u_1 + 5 \\ &= 3a + 5 \end{aligned}$$

$$\begin{aligned} u_3 &= 3u_2 + 5 \\ &= 3(3a + 5) + 5 \\ &= 9a + 20 \end{aligned}$$

$$\begin{aligned} \textcircled{b} \quad u_4 &= 3u_3 + 5 \\ &= 3(9a + 20) + 5 \\ &= 27a + 65 \end{aligned}$$

$$\begin{aligned} \sum_{r=1}^4 u_r &= u_1 + u_2 + u_3 + u_4 \\ &= a + (3a + 5) + (9a + 20) + (27a + 65) \\ &= 40a + 90 \end{aligned}$$

$$\begin{aligned} \text{Since } 40a + 90 &= 110 \\ 40a &= 20 \\ a &= \frac{1}{2} \end{aligned}$$



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- 2 (a) The sum of the first 20 terms of an arithmetic sequence is 65. The sum of the next 8 terms is -30 .

Find the first term and the common difference of this sequence.

[3]

- (b) A sequence has n th term given by $u_n = e^{k(n+1)}$, where k is a constant and $k \neq 0$.

Show that this sequence is geometric.

[2]

$$\begin{aligned} \textcircled{a} \quad \frac{20}{2} [2a + 19d] &= 65 & S_{28} - S_{20} &= -30 \\ 20a + 190d &= 65 & \frac{28}{2} [2a + 27d] - 65 &= -30 \\ 4a + 38d &= 13 \text{ --- } \textcircled{1} & 28a + 378d &= 35 \text{ --- } \textcircled{2} \end{aligned}$$

$$\textcircled{1} \times 7: 28a + 266d = 91 \text{ --- } \textcircled{3}$$

$$\begin{aligned} \textcircled{3} - \textcircled{2}: -112d &= 56 \\ d &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{Sub } d = -\frac{1}{2} \text{ into } \textcircled{1}: 4a + 38\left(-\frac{1}{2}\right) &= 13 \\ 4a &= 32 \\ a &= 8 \end{aligned}$$

$$\begin{aligned} \textcircled{b} \quad \frac{u_n}{u_{n-1}} &= \frac{e^{k(n+1)}}{e^{k(n-1+1)}} = e^{kn+k-kn} \\ &= e^k \\ &= \text{constant} \end{aligned}$$

Since $\frac{u_n}{u_{n-1}} = \text{constant}$, this sequence is geometric with a common ratio of e^k .

3 Do not use a calculator in answering this question.

A complex number z is given by $z = \frac{\sqrt{3} + i}{\sqrt{3} - i}$.

(a) Find $|z|$ and $\arg(z)$.

[4]

The point A represents the complex number z on an Argand diagram. The point A is rotated anticlockwise through $\frac{1}{2}\pi$ about the origin O to the point B .

(b) Plot the points A and B on an Argand diagram. State in terms of z the complex number that is represented by B , explaining your answer.

[3]

$$\textcircled{a}. \quad z = \frac{\sqrt{3} + i}{\sqrt{3} - i} \times \frac{\sqrt{3} + i}{\sqrt{3} + i}$$

$$= \frac{3 + 2\sqrt{3}i - 1}{3 + 1}$$

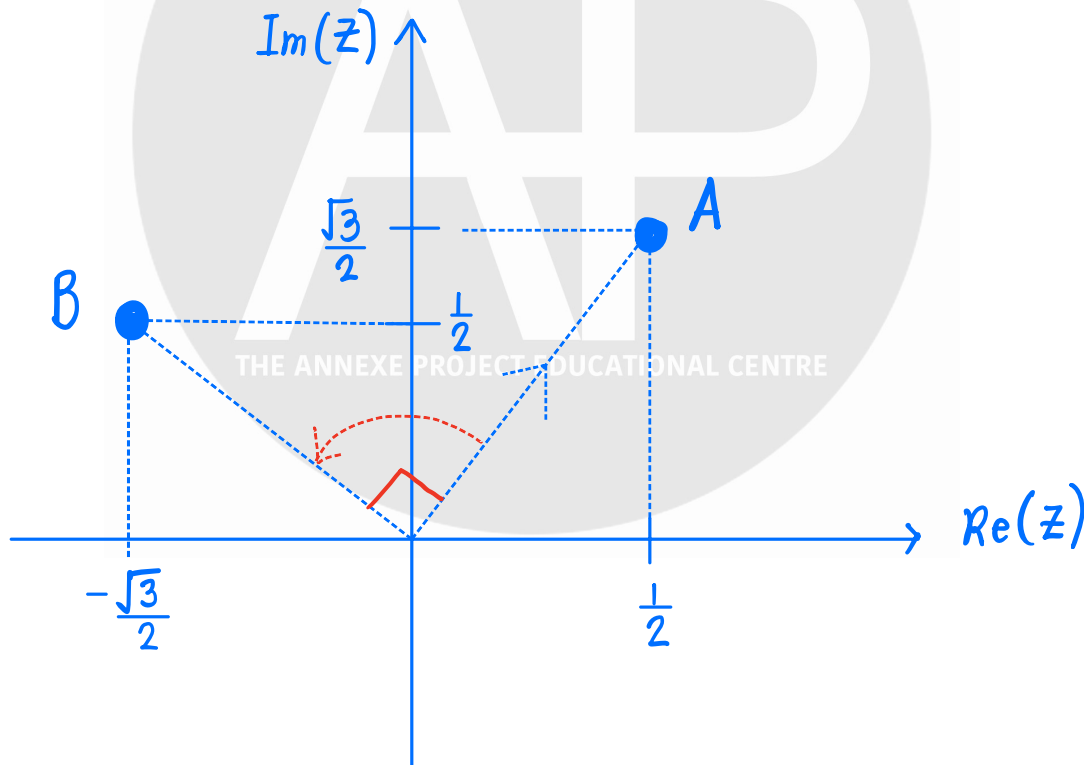
$$= \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$|z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= 1 \text{ unit}$$

$$\arg(z) = \tan^{-1} \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{\pi}{3}$$

\textcircled{b}.



$B = iz$, as multiplication of any complex number represents an anticlockwise rotation of $\frac{\pi}{2}$ about O .

4 A curve is given by the equation $y = \frac{3x+10}{\sqrt{x+1}}$.

(a) Find $\frac{dy}{dx}$ and hence find the x -coordinate of the turning point of the curve. [3]

(b) Use calculus to find the value of the second derivative at the turning point and state the nature of this turning point. [3]

$$\begin{aligned}\textcircled{a} \quad \frac{dy}{dx} &= \frac{(x+1)^{1/2}(3) - (3x+10) \cdot \frac{1}{2}(x+1)^{-1/2}}{(x+1)} \\&= \frac{\frac{1}{2}(x+1)^{-1/2} [6(x+1) - (3x+10)]}{(x+1)} \\&= \frac{(3x-4)}{2(x+1)^{3/2}}\end{aligned}$$

$$\text{Let } \frac{dy}{dx} = 0 : \quad \begin{aligned}3x-4 &= 0 \\x &= \frac{4}{3}\end{aligned}$$

$$\begin{aligned}\textcircled{b} \quad \frac{d^2y}{dx^2} &= \frac{2(x+1)^{3/2}(3) - (3x-4) \cdot 3(x+1)^{1/2}}{4(x+1)^3} \\&= \frac{3(x+1)^{1/2} [2(x+1) - (3x-4)]}{4(x+1)^3} \\&= \frac{3(6-x)}{4(x+1)^{5/2}}\end{aligned}$$

$$\text{When } x = \frac{4}{3}, \quad \frac{d^2y}{dx^2} = \frac{14}{4\left(\frac{7}{3}\right)^{5/2}} = 0.421 > 0$$

By 2nd Derivative Test, it is a minimum point.

5 (a) Solve exactly the inequality $e^x \geq 2e^{-x} + 1$.

[3]

(b) Hence find $\int_0^1 |e^x - 2e^{-x} - 1| dx$, giving your answer in exact form.

[4]

① Let $u = e^x$:

$$u \geq \frac{2}{u} + 1$$

Since $u > 0$, we can multiply both sides by u without the need to change the inequality sign.

$$u^2 \geq 2 + u$$

$$u^2 - u - 2 \geq 0$$

$$(u+1)(u-2) \geq 0$$

$$u \leq -1 \text{ or } u \geq 2$$

$$\therefore e^x \leq -1 \quad \text{or} \quad e^x \geq 2$$

(No Solution as $e^x > 0$)

$$x \geq \ln 2$$

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②

$$\int_0^1 |e^x - 2e^{-x} - 1| dx$$

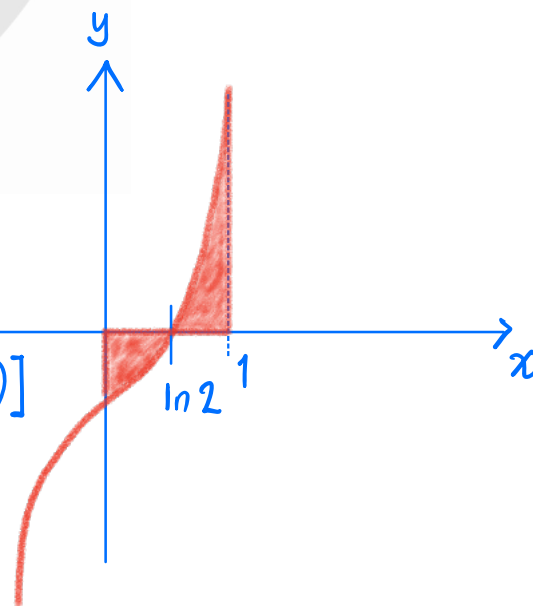
$$= \int_0^{\ln 2} -(e^x - 2e^{-x} - 1) dx + \int_{\ln 2}^1 (e^x - 2e^{-x} - 1) dx$$

$$= -[e^x + 2e^{-x} - x]_0^{\ln 2} + [e^x + 2e^{-x} - x]_{\ln 2}^1$$

$$= -[(2 + 1 - \ln 2) - (1 + 2)] + [(e + \frac{2}{e} - 1) - (2 + 1 - \ln 2)]$$

$$= \ln 2 + e + \frac{2}{e} - 4 + \ln 2$$

$$= (2\ln 2 + e + \frac{2}{e} - 4) \text{ sq. units.}$$



- 6 The carbon content of plants includes a small proportion of the radioactive isotope carbon-14. The mass of carbon-14 in dead plants decreases over time. Scientists measure the mass of carbon-14 in plants to estimate how long ago they died.

The mass of carbon-14 in dead plants decreases by 50% in 5730 years.

The mass, M , of carbon-14 in a plant t years after it has died is modelled by the differential equation

$$\frac{dM}{dt} = -kM,$$

where k is a positive constant. The initial value of M is M_0 .

- (a) By solving the differential equation, find the value of k . [4]
- (b) A dead plant has 20% of its original mass of carbon-14 remaining. Determine, to the nearest 100 years, how long ago the plant died. [2]

(a) Given $\frac{dM}{dt} = -kM$

$$\int \frac{1}{M} dM = -k \int dt$$

$$\ln |M| = -kt + C$$

$$\ln M = -kt + C \quad (\text{Since } M \geq 0)$$

$$M = e^{-kt+C}$$

$$= e^{-kt} e^C$$

$$= A e^{-kt} \quad (\text{where } A = e^C)$$

When $t=0$, $M=M_0$: $M_0 = A$

$$\therefore M = M_0 e^{-kt}$$

When $t=5730$, $M = \frac{1}{2} M_0$:

$$\frac{1}{2} M_0 = M_0 e^{-5730k}$$

$$\ln \frac{1}{2} = -5730k$$

$$\ln 1 - \ln 2 = -5730k$$

$$-\ln 2 = -5730k$$

$$k = \frac{\ln 2}{5730}$$

(b) When $M = 0.2 M_0$: $0.2 M_0 = M_0 e^{-\frac{\ln 2}{5730} t}$

$$\ln 0.2 = -\frac{\ln 2}{5730} t$$

$$t = 13304.65$$

$$= 13300 \text{ years (nearest 100)}$$

- 7 (a) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$. [2]

A curve C is given by the equation $y = \ln x$. The region R is bounded by C , the x -axis and the line $x = e$.

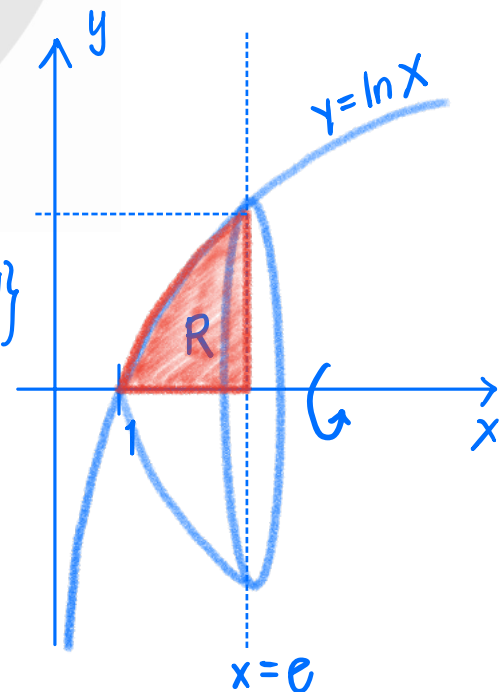
- (b) Using the result in part (a), find the exact volume generated when R is rotated 360° about the x -axis. [4]
- (c) Find the volume generated when R is rotated 360° about the y -axis, giving your answer correct to 2 decimal places. [3]

$$\begin{aligned} \textcircled{a} \quad \frac{d}{dx}(x \ln x - x) &= x \left(\frac{1}{x} \right) + \ln x - 1 \\ &= 1 + \ln x - 1 \\ &= \ln x \end{aligned}$$

Using By Parts :

$$\begin{array}{ll} \text{Let } u = \ln x & \text{Let } dv = \ln x \\ \frac{du}{dx} = \frac{1}{x} & v = x \ln x - x \end{array}$$

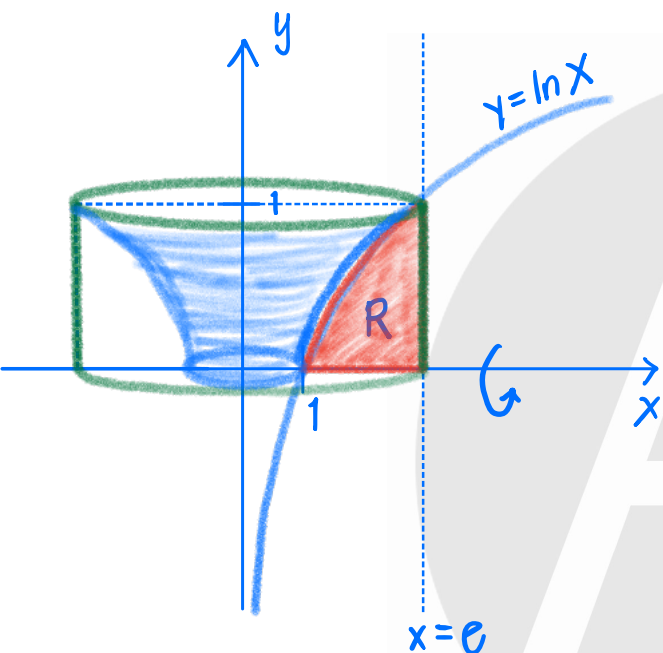
$$\begin{aligned} \textcircled{b} \quad V_x &= \pi \int_1^e (\ln x)^2 dx \\ &= \pi \int_1^e (\ln x)(\ln x) dx \\ &= \pi \left[(\ln x)(x \ln x - x) - \int_1^e \frac{1}{x} (x \ln x - x) dx \right]_1^e \\ &= \pi \left[(\ln x)(x \ln x - x) - \int_1^e \ln x - 1 dx \right]_1^e \\ &= \pi \left[(\ln x)(x \ln x - x) - (x \ln x - x) + x \right]_1^e \\ &= \pi \left[(x \ln x - x)(\ln x - 1) + x \right]_1^e \\ &= \pi \left\{ [(e - e)(1 - 1) + e] - [(0 - 1)(0 - 1) + 1] \right\} \\ &= \pi (e - 2) \text{ cubic units.} \end{aligned}$$



For $y = \ln x$: Make x the subject.
 $x = e^y$

© By GC:

$$\begin{aligned}
 V_y &= \text{Volume of green cylinder} - \text{blue Volume} \\
 &= \pi(e^2)(1) - \pi \int_0^1 (e^y)^2 dy \\
 &= 13.177 \\
 &= 13.18 \text{ cubic units (2 d.p.)}
 \end{aligned}$$



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8 The plane π_1 has equation $2x + y + 3z = 2$. A point P has position vector $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$.

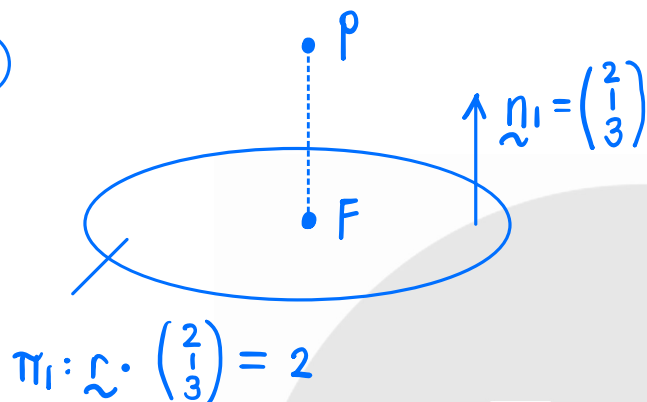
(a) Find the foot of the perpendicular from P to the plane π_1 . [4]

(b) Find the position vector of the reflection of P in the plane π_1 . [2]

A second plane π_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$, where λ and μ are parameters.

(c) Find the acute angle between π_1 and π_2 . [3]

(a)



Let F be the foot of perpendicular from P to π_1 .

Step 1: Create line PF

$$l_{\overrightarrow{PF}}: \mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \lambda \in \mathbb{R}$$

Step 2:

$\overrightarrow{OF}$ lies on $l_{\overrightarrow{PF}}$,

$$\text{hence } \overrightarrow{OF} = \begin{pmatrix} -3 + 2\lambda \\ 1 + \lambda \\ 3\lambda \end{pmatrix}$$

Step 3:

Sub $\overrightarrow{OF}$ into eqn of π_1 .

$$\begin{pmatrix} -3 + 2\lambda \\ 1 + \lambda \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 2$$

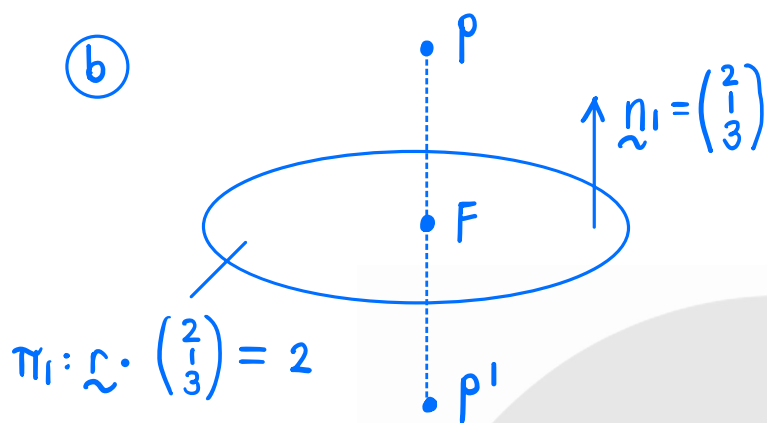
$$-6 + 4\lambda + 1 + \lambda + 9\lambda = 2$$

$$14\lambda = 7$$

$$\lambda = \frac{1}{2}$$

$$\therefore \overrightarrow{OF} = \begin{pmatrix} -3 + 2 \times \frac{1}{2} \\ 1 + \frac{1}{2} \\ 3 \times \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -2 \\ 1.5 \\ 1.5 \end{pmatrix}$$

⑥



Using Ratio Theorem:

$$\vec{OF} = \frac{\vec{OP} + \vec{OP'}}{2}$$

$$\begin{aligned} \vec{OP'} &= 2\vec{OF} - \vec{OP} \\ &= 2\begin{pmatrix} -2 \\ 1.5 \\ 1.5 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \end{aligned}$$

⑦ $\vec{n}_2 = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -13 \\ 4 \\ 6 \end{pmatrix}$

Acute angle between π_1 and $\pi_2 = \cos^{-1} \left| \frac{\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -13 \\ 4 \\ 6 \end{pmatrix}}{\sqrt{14} \sqrt{221}} \right|$

$$= \cos^{-1} \left| \frac{-4}{\sqrt{14} \sqrt{221}} \right|$$

$$= 85.9^\circ$$



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9 It is given that $y = \sin^{-1} px$, where p is a constant.

(a) Show that $\frac{dy}{dx} = p \sec y$. [2]

(b) By further differentiation of the result in part (a) find, in terms of p , the Maclaurin series for y up to and including the term in x^3 . [5]

(c) Use your answer to part (b) to find the series expansion of $\frac{1}{\sqrt{1-9x^2}}$, up to and including the term in x^2 . [3]

① $y = \sin^{-1} px$

$$\sin y = px$$

differentiate both sides w.r.t. x :

$$\cos y \cdot \frac{dy}{dx} = p$$

$$\frac{dy}{dx} = \frac{p}{\cos y} = p \sec y \quad (\text{shown}).$$

② $\frac{d^2y}{dx^2} = p \sec y \tan y \frac{dy}{dx}$
 $= \frac{dy}{dx} \cdot \tan y \cdot \frac{dy}{dx}$
 $= (\tan y) \left(\frac{dy}{dx} \right)^2$

$$\frac{d^3y}{dx^3} = (\tan y) \cdot 2 \left(\frac{dy}{dx} \right) \left(\frac{d^2y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2 \cdot \sec^2 y \frac{dy}{dx}$$
$$= 2 \tan y \left(\frac{dy}{dx} \right) \left(\frac{d^2y}{dx^2} \right) + \sec^2 y \left(\frac{dy}{dx} \right)^3$$

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When $x = 0$:

$$y = \sin^{-1} 0 = 0,$$

$$\frac{dy}{dx} = p \sec 0 = \frac{p}{\cos 0} = p,$$

$$\frac{d^2y}{dx^2} = (\tan 0) \cdot p^2 = 0,$$

$$\frac{d^3y}{dx^3} = \sec^2 0 \cdot p^3 = p^3$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \dots + \frac{f^{(r)}(0)}{r!}x^r$$

$$\text{By Maclaurin series, } \sin^{-1} px = 0 + px + 0x^2 + \frac{p^3}{3!}x^3 + \dots$$
$$= px + \frac{1}{6}p^3x^3 + \dots$$

© Let $p = 3$: $\sin^{-1} 3x \approx 3x + \frac{9}{2}x^3$

Differentiate both sides w.r.t. x :

$$\frac{1}{\sqrt{1-(3x)^2}} \cdot 3 = 3 + \frac{9}{2}(3x^2)$$

$$\frac{3}{\sqrt{1-9x^2}} = 3 + \frac{27}{2}x^2$$

$$\therefore \frac{1}{\sqrt{1-9x^2}} = 1 + \frac{9}{2}x^2$$

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10 The function f is given by

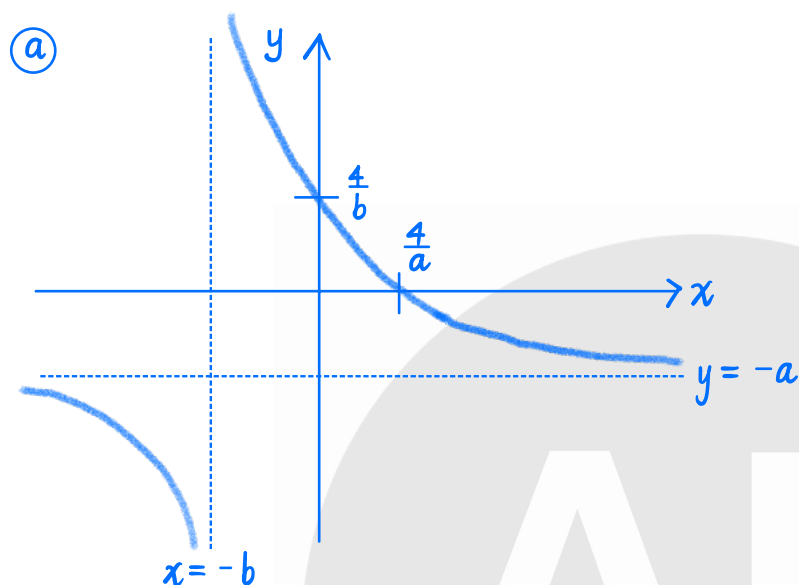
$$f: x \mapsto \frac{4-ax}{x+b}, \quad x \in \mathbb{R}, \quad x \neq -b,$$

where a and b are positive constants.

- (a) Sketch the graph of $y = f(x)$, stating the equations of any asymptotes and the coordinates of any points where the curve crosses the axes. [4]
- (b) Describe a sequence of 3 transformations that would transform the graph of $y = f(x)$ onto the graph of $y = \frac{1}{x}$. [4]
- (c) Find $f^{-1}(x)$ and state its range. [3]

It is now given that $b = a$.

- (d) Find the value of the composite function $f^{2025}(x)$ when $x = 2$, giving your answer in terms of a . [1]



$$\begin{array}{r} -a \\ x+b \overline{) -ax + 4} \\ \underline{-(-ax - ab)} \\ 4+ab \end{array}$$

$$\therefore y = -a + \frac{4+ab}{x+b}$$

Horizontal asymptote:

$$\begin{array}{l} \text{As } x \rightarrow \pm \infty, \\ y \rightarrow -a \end{array}$$

(b) $y = -a + \frac{4+ab}{x+b}$

↓ replace y with $y-a$
(translation of a units along positive y -axis)

$$y = \frac{4+ab}{x+b}$$

↓ replace x with $x-b$
(translation of b units along positive x -axis)

$$y = \frac{4+ab}{x}$$

↓ replace x with $(4+ab)x$
(scaling // x -axis by factor $\frac{1}{4+ab}$)

$$y = \frac{1}{x}$$

$$\textcircled{c} \quad y = \frac{4-ax}{x+b}$$

$$xy + by = 4 - ax$$

$$xy + ax = 4 - by$$

$$x = \frac{4-by}{y+a}$$

$$\therefore f^{-1}(x) = \frac{4-bx}{x+a}$$

$$R_{f^{-1}} = D_f = (-\infty, -b) \cup (-b, \infty)$$

$$\textcircled{d} \quad \text{If } a=b, \quad f^{-1}(x) = f(x) \quad [\text{self-inverse functions}]$$

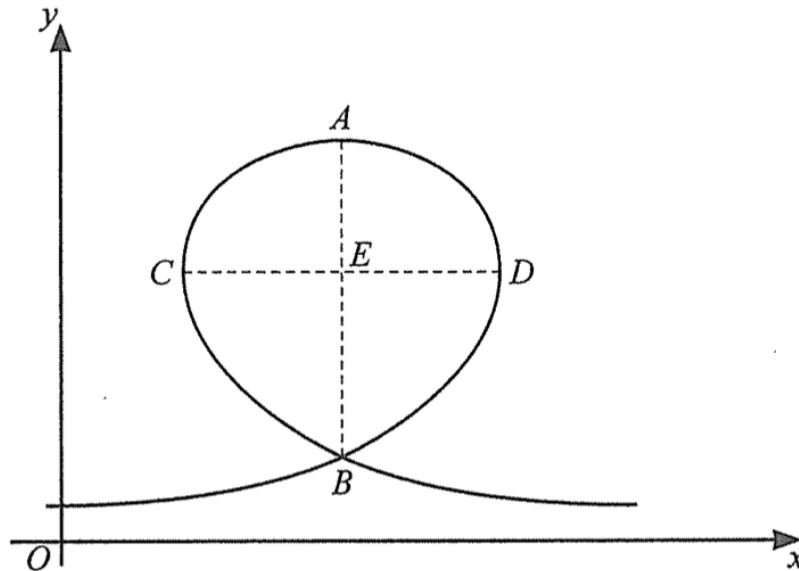
$$f^{2025}(2) = f(2) = \frac{4-a(2)}{2+a}$$

$$= \frac{4-2a}{2+a}$$



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Engineers are designing a roller coaster. The diagram shows the side view of a section of track with a loop. This section is modelled by the curve with parametric equations

$$x = 28(\theta \cos \theta + 1), \quad y = 20(2 - \theta \sin \theta),$$

for $-\frac{2}{3}\pi \leq \theta \leq \frac{2}{3}\pi$, where x and y are measured in metres. The x -axis models the horizontal ground.

The maximum point on the curve is at A , and the line AB is vertical. Points C and D are where the tangent to the curve is parallel to the y -axis. Point E is the intersection of the lines AB and CD .

(a) Find $\frac{dy}{dx}$ in terms of θ . [3]

(b) (i) Find the values of θ at points C and D , giving your answers correct to 2 significant figures. [3]

(ii) Hence find the greatest width, CD , of the loop. [2]

$$\begin{aligned} \textcircled{a} \quad \frac{dx}{d\theta} &= 28[\theta(-\sin \theta) + \cos \theta(1)] \\ &= 28[\cos \theta - \theta \sin \theta] \end{aligned}$$

$$\begin{aligned} \frac{dy}{d\theta} &= 20[-\theta(\cos \theta) + \sin \theta(-1)] \\ &= -20[\theta \cos \theta + \sin \theta] \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{-20(\theta \cos \theta + \sin \theta)}{28(\cos \theta - \theta \sin \theta)} \\ &= \frac{-5(\theta \cos \theta + \sin \theta)}{7(\cos \theta - \theta \sin \theta)} \end{aligned}$$

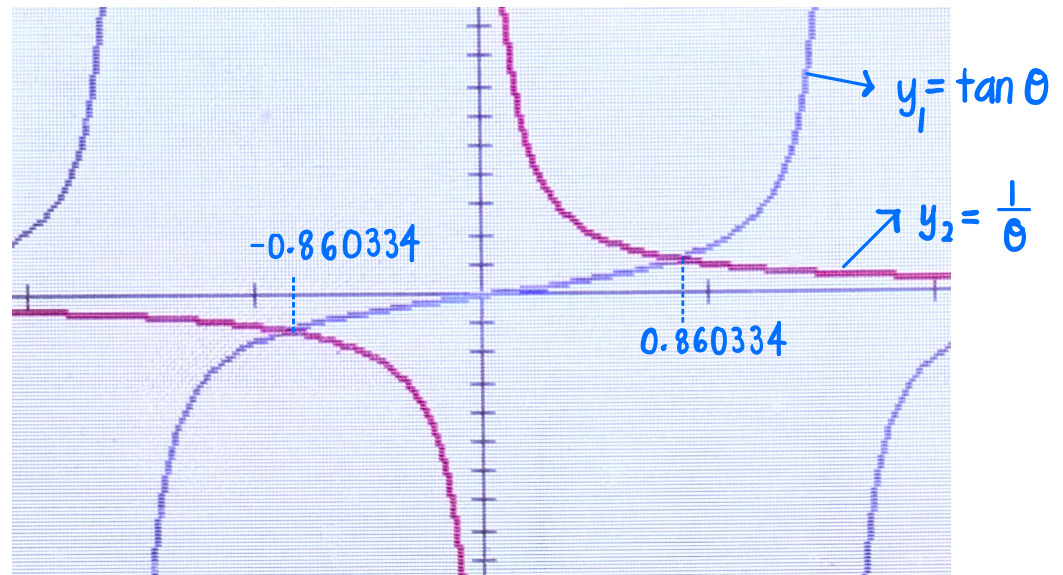
⑥ i. Let $\frac{dy}{dx}$ be undefined.

$$\therefore 7(\cos \theta - \theta \sin \theta) = 0$$

$$\cos \theta = \theta \sin \theta \quad \left(\begin{array}{l} \text{divide both sides by } \cos \theta \\ 1 = \theta \tan \theta \end{array} \right)$$

$$\tan \theta = \frac{1}{\theta}$$

By GC:



$$\text{At } C, \theta = -0.86$$

$$\text{At } D, \theta = 0.86$$

$$\begin{aligned} \text{ii. } x\text{-coordinate of } C &= 28(-0.86 \cos -0.86 + 1) \\ &= 12.289 \end{aligned}$$

$$\begin{aligned} x\text{-coordinate of } D &= 28(0.86 \cos 0.86 + 1) \\ &= 43.711 \end{aligned}$$

$$CD = 43.711 - 12.289 = 31.4 \text{ m}$$

The x-coordinate of point A is 28.

(c) (i) Find the value of θ at point A. [1]

(ii) Find the 2 possible values of θ at point B. [2]

(iii) Hence find the distance AB. [1]

Engineers know that, for safety reasons, the distance AE should be within 10% of the distance $\frac{1}{2}CD$.

(d) Determine whether the design of this section of track satisfies this condition. [2]

③ i. Let $28(\theta \cos \theta + 1) = 28$

$$\theta \cos \theta + 1 = 1$$

$$\theta \cos \theta = 0$$

$$\theta = 0 \quad \text{or} \quad \cos \theta = 0$$

$$\theta = -\frac{\pi}{2} \quad \text{or} \quad \frac{\pi}{2} \quad \left[\text{because } -\frac{2\pi}{3} \leq \theta \leq \frac{2\pi}{3} \right]$$

At $\theta = 0$, $y = 20(2 - 0)$
 $= 40$ (y-coordinate of A)

At $\theta = -\frac{\pi}{2}$, $y = 20(2 + \frac{\pi}{2}(-1))$
 $= 8.5841$

At $\theta = \frac{\pi}{2}$, $y = 20(2 - \frac{\pi}{2}(1))$
 $= 8.5841$ (y-coordinate of B)

$\therefore$ at point A, $\theta = 0$ (because A is at a higher point than B)

ii. at point B, $\theta = -\frac{\pi}{2}$ or $\frac{\pi}{2}$

iii. $AB = 40 - 8.5841 = 31.4 \text{ m}$

④ y-coordinate of C = $20(2 - (-0.860334) \sin(-0.860334))$
 $= 26.956$

distance AE = $40 - 26.956 = 13.044 \text{ m}$

distance CD = 31.4 m

Within 10% of $\frac{1}{2}CD = (15.7 \pm 1.57) \text{ m}$

For safety reasons, $14.13 \leq AE \leq 17.27$.

Since $AE = 13.044 \text{ m}$, the design of this section of track does not satisfy the safety conditions.

12 Do not use a calculator in answering this question.

It is given that $1+2i$ is a complex root of the equation $z^3 + pz^2 + qz - 10 = 0$, where p and q are real numbers.

- (a) State another complex root of the equation in the form $a+ib$, where a and b are real numbers and $b \neq 0$. Justify your answer. [2]
- (b) Find the values of p and q , and find the third root. [6]
- (c) Hence find the roots of the equation $1 + pw + qw^2 - 10w^3 = 0$. [2]

① Since $p, q \in \mathbb{R}$, all coefficients of the cubic polynomial are real. By Complex Conjugate Root Theorem, $\bar{z} = 1-2i$ is another root of this equation.

$$\begin{aligned}\textcircled{b} [z - (1+2i)][z - (1-2i)] &= z^2 - z(1-2i) - z(1+2i) + (1+2i)(1-2i) \\ &= z^2 - 2z + (1 - 4i^2) \\ &= z^2 - 2z + 5\end{aligned}$$

Let the third root be a , $a \in \mathbb{R}$:

$$\begin{aligned}z^3 + pz^2 + qz - 10 &= (z^2 - 2z + 5)(z - a) \\ &= z^3 + (-a-2)z^2 + (2a+5)z - 5a\end{aligned}$$

By comparison of coefficients,

$$-10 = -5a$$

$$a = 2 \quad (\text{3rd root is } 2)$$

$$\begin{aligned}p &= -a-2 \\ &= -2-2 \\ &= -4\end{aligned}$$

$$\begin{aligned}q &= 2a+5 \\ &= 4+5 \\ &= 9\end{aligned}$$

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② Replace z with $\frac{1}{w}$:

$$\begin{aligned}\therefore \frac{1}{w} &= 1+2i, & \frac{1}{w} &= 1-2i, & \frac{1}{w} &= 2 \\ w &= \frac{1}{1+2i} \times \frac{1-2i}{1-2i}, & w &= \frac{1}{1-2i} \times \frac{1+2i}{1+2i}, & w &= \frac{1}{2} \\ &= \frac{1-2i}{1+4}, & &= \frac{1+2i}{5} \\ &= \frac{1}{5}(1-2i), & &= \frac{1}{5}(1+2i)\end{aligned}$$

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(2025)6
H2 Mathematics