

MATHEMATICS

Paper 2

9758/02

October/November 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

- 1 In a game, players have to collect tokens in the form of cars, trains and boats. A car is worth c points, a train t points and a boat b points. At the end of a game the total score of each player is worked out and the player with the highest score wins.

Pritti, Quentin, Ria and Sam played a game. At the end of the game:

- Pritti had 8 cars, 11 trains and 5 boats and scored 114 points.
- Quentin had 5 cars, 14 trains and 7 boats and scored 112 points.
- Ria had 9 cars, 9 trains and 4 boats and scored 110 points.
- Each player scored a different number of points.
- Sam came second.

(a) Find the values of c , t and b .

[3]

At the end of the game, Sam had at least 6 of each type of token, different numbers of each type of token, and more boats than trains.

(b) Determine how many of each type of token Sam had at the end of the game. [You are given that there is only one solution.]

[3]

$$\begin{aligned} \textcircled{a} \quad 8c + 11t + 5b &= 114 \quad \text{---} \textcircled{1} \\ 5c + 14t + 7b &= 112 \quad \text{---} \textcircled{2} \\ 9c + 9t + 4b &= 110 \quad \text{---} \textcircled{3} \end{aligned}$$

By GC: $c = 7, t = 3, b = 5$

$\textcircled{b}$ Since Sam came in second, his score is 113.
Let $a+6$, $b+6$, and $c+6$ be the number of cars, trains and boats Sam have.

$$7(a+6) + 3(b+6) + 5(c+6) = 113$$

$$7a + 42 + 3b + 18 + 5c + 30 = 113$$

$$7a + 3b + 5c = 23$$

Since there is only one solution, and the number of each token is different, by observation

$$a = 0, b = 1, c = 4$$

Sam has 6 cars, 7 trains and 10 boats.

- 2 The points P , Q and R are collinear. P and Q have position vectors $\begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix}$ respectively. The point R is such that $3\overrightarrow{QR} = 4\overrightarrow{PQ}$.

(a) Find the position vector of R .

[3]

The point S has position vector $\begin{pmatrix} -1 \\ -2 \\ c \end{pmatrix}$, where c is a constant and $\overrightarrow{SQ}$ is perpendicular to $\overrightarrow{PQ}$.

(b) Find the value of c .

[2]

(c) Use a scalar product to find the angle between $\overrightarrow{PS}$ and $\overrightarrow{PQ}$.

[3]

$$\textcircled{a} \quad 3[\overrightarrow{OR} - \overrightarrow{OQ}] = 4[\overrightarrow{OQ} - \overrightarrow{OP}]$$

$$3\overrightarrow{OR} = 7\overrightarrow{OQ} - 4\overrightarrow{OP}$$

$$\begin{aligned} \overrightarrow{OR} &= \frac{1}{3} \left[7 \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - 4 \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} \right] \\ &= \begin{pmatrix} 5 \\ 10 \\ 14 \end{pmatrix} \end{aligned}$$

$$\textcircled{b} \quad \overrightarrow{SQ} = \overrightarrow{OQ} - \overrightarrow{OS} = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 6-c \end{pmatrix}$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$$

$$\overrightarrow{SQ} \cdot \overrightarrow{PQ} = 0$$

$$\begin{pmatrix} 2 \\ 8 \\ 6-c \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} = 0$$

$$6 + 24 + 36 - 6c = 0$$

$$6c = 66$$

$$c = 11$$

$$\textcircled{c} \quad \overrightarrow{PS} = \overrightarrow{OS} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ -2 \\ 11 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix}$$

$$\theta = \cos^{-1} \frac{\begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}}{\sqrt{147} \sqrt{54}}$$

$$= \cos^{-1} \frac{54}{\sqrt{147} \sqrt{54}}$$

$$= 52.7^\circ$$



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3 The function $y = f(x)$ is such that $\frac{dy}{dx} = y^2 \sin 3x$, and $y = 1$ when $x = \pi$.

(a) Show that $f(x) = \frac{A}{B + \cos 3x}$, where A and B are constants to be determined. [4]

(b) (i) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 2\pi$. State the values of c for which the line $y = c$ is a tangent to the curve. [3]

(ii) State the equations of the axes of symmetry of $y = f(x)$. [2]

(iii) Find an expression for $f(x + \pi)$, giving your answer in terms of $\cos 3x$. [1]

(c) Show that $\frac{d^2y}{dx^2} = Py^2 + Qy^3$, where P and Q are functions of x to be determined. [3]

a) $\frac{dy}{dx} = y^2 \sin 3x$

$$\int \frac{1}{y^2} dy = \int \sin 3x dx$$

$$-\frac{1}{y} = -\frac{\cos 3x}{3} + C$$

Given $y = 1$ when $x = \pi$:

$$-1 = -\frac{\cos 3\pi}{3} + C$$

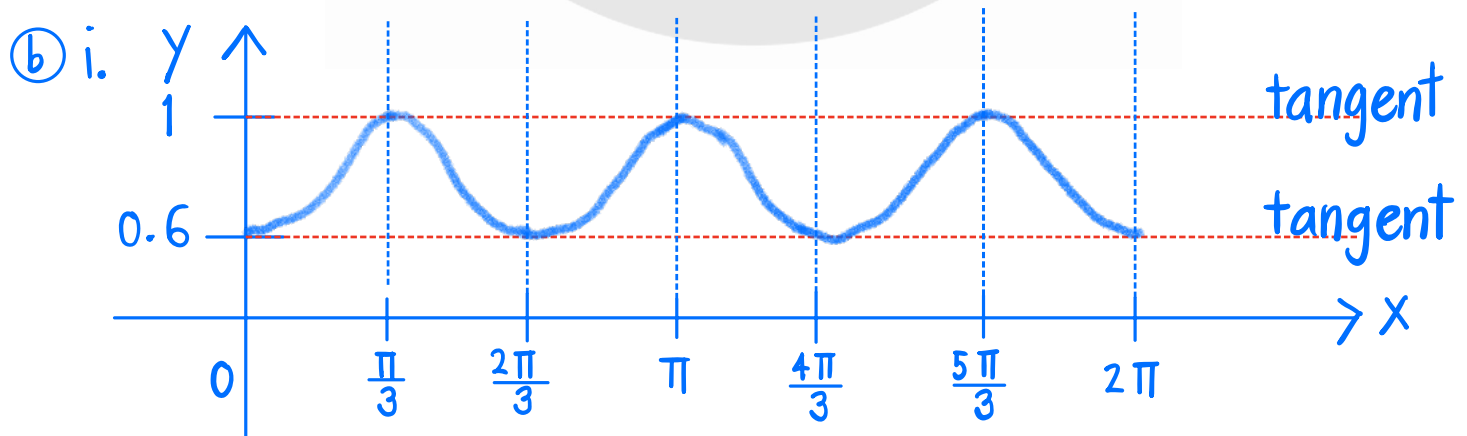
$$-1 = \frac{1}{3} + C$$

$$C = -\frac{4}{3}$$

$$\therefore -\frac{1}{y} = -\frac{\cos 3x}{3} - \frac{4}{3}$$

$$\frac{1}{y} = \frac{\cos 3x + 4}{3}$$

$$y = \frac{3}{4 + \cos 3x} \quad \text{where } A = 3, B = 4$$



$c = 0.6$ and 1

ii. Possible axes of symmetry :

$$x = \frac{\pi}{3}, \quad x = \frac{2\pi}{3}, \quad x = \frac{3\pi}{3}, \dots$$

$$\text{i.e. } x = \frac{k\pi}{3}, \text{ where } k \in \mathbb{Z}$$

$$\begin{aligned} \text{iii. } f(x + \pi) &= \frac{3}{4 + \cos 3(x + \pi)} \\ &= \frac{3}{4 + \cos(3x + 3\pi)} \\ &= \frac{3}{4 + \cos 3x \cos 3\pi - \sin 3x \sin 3\pi} \\ &= \frac{3}{4 - \cos 3x} \end{aligned}$$

③ $\frac{dy}{dx} = y$

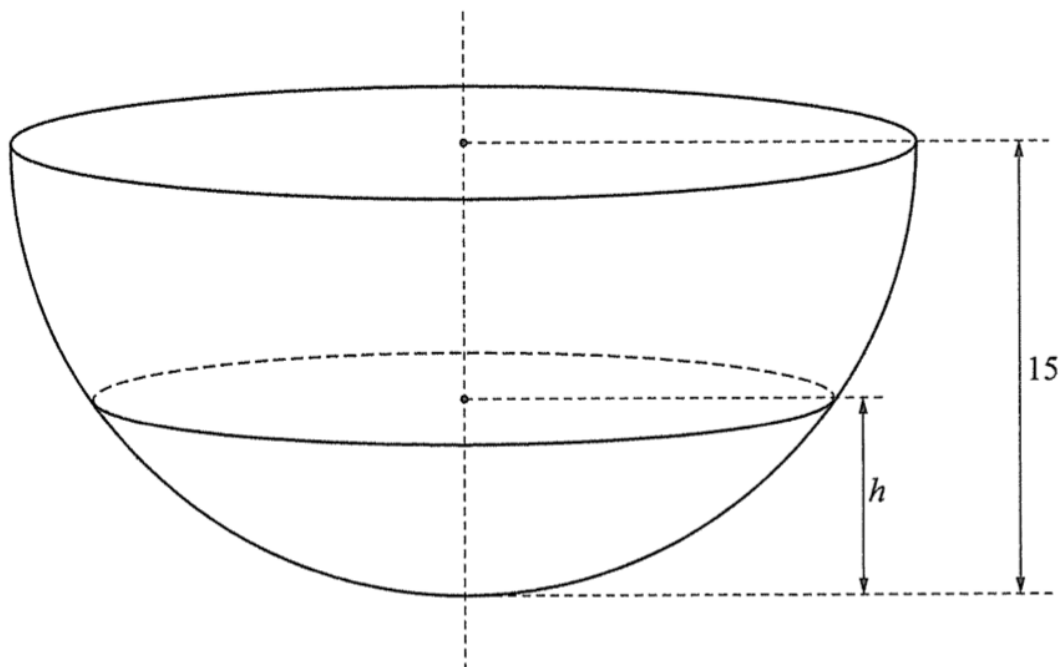
$$\begin{aligned} \frac{d^2y}{dx^2} &= y^2 \cdot 3 \cos 3x + \sin 3x \cdot 2y \frac{dy}{dx} \\ &= 3 \cos 3x \cdot y^2 + 2 \sin 3x \cdot y (y^2 \sin 3x) \\ &= (3 \cos 3x) y^2 + (2 \sin^2 3x) y^3 \end{aligned}$$

$$\text{where } P = 3 \cos 3x, \quad Q = 2 \sin^2 3x$$



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The diagram shows a hemispherical bowl, of inside radius 15 cm, fixed so its circular rim is horizontal. When the depth of water in the bowl is h cm, the volume, $V \text{ cm}^3$, of water in the bowl is given by $V = \frac{1}{3}\pi h^2(45 - h)$.

The bowl is initially empty, and water is poured into the bowl at the constant rate of $10\pi \text{ cm}^3$ per second.

- (a) Find the time taken to fill the bowl. [2]
- (b) Find the exact rate at which the depth of water is increasing when the depth of water is 12 cm. [4]

$$\textcircled{a} \text{ Vol. of bowl} = \frac{\pi}{3}(15)^2(45-15)$$

$$= 2250\pi \text{ cm}^3$$

$$\text{time taken to fill the bowl} = \frac{2250\pi}{10\pi} = 225 \text{ s}$$

$$\textcircled{b} \quad V = \frac{\pi}{3}h^2(45-h)$$

$$= 15\pi h^2 - \frac{\pi}{3}h^3$$

$$\frac{dV}{dh} = 30\pi h - \pi h^2$$

$$\text{Given } \frac{dV}{dt} = 10\pi \text{ cm}^3/\text{s}$$

$$\text{Find } \frac{dh}{dt} \text{ when } h = 12 \text{ cm}$$

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$$10\pi = [30\pi(12) - \pi(12)^2] \cdot \frac{dh}{dt}$$

$$10\pi = (360\pi - 144\pi) \cdot \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{5}{108} \text{ cm/s}$$



The bowl is emptied, and water is poured into the bowl at the variable rate of $10\pi t \text{ cm}^3$ per second, where t seconds is the time from when pouring started.

(c) Find the time taken to pour $972\pi \text{ cm}^3$ of water into the bowl. [3]

(d) Find the rate at which the depth of water is increasing when the volume of water in the bowl is $972\pi \text{ cm}^3$. [4]

③ Given $\frac{dV}{dt} = 10\pi t \text{ cm}^3/\text{s}$

$$V = 10\pi \int t \, dt$$

$$= \frac{10\pi t^2}{2}$$

$$= 5\pi t^2$$

When $V = 972\pi$, $972\pi = 5\pi t^2$

$$t = \sqrt{\frac{972}{5}}$$

$$= 13.9 \text{ s}$$

④ Step 1 :

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$$10\pi t = [30\pi h - \pi h^2] \cdot \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{10\pi t}{(30\pi h - \pi h^2)}$$

Step 2 :

$$V = \frac{\pi}{3} h^2 (45 - h)$$

When $V = 972\pi$, $972\pi = \frac{\pi}{3} h^2 (45 - h)$

$$2916 = 45h^2 - h^3$$

$$h^3 - 45h^2 + 2916 = 0$$

By GC: $h = -7.4558$, $h = 9$ or $h = 43.456$
(rej.) (rej.)

Step 3 :

when $h = 9 \text{ cm}$, $t = \sqrt{\frac{972}{5}}$

$$\frac{dh}{dt} = \frac{10\pi \sqrt{\frac{972}{5}}}{[30\pi(9) - \pi(9)^2]} = 0.73771 = 0.738 \text{ cm/s}$$

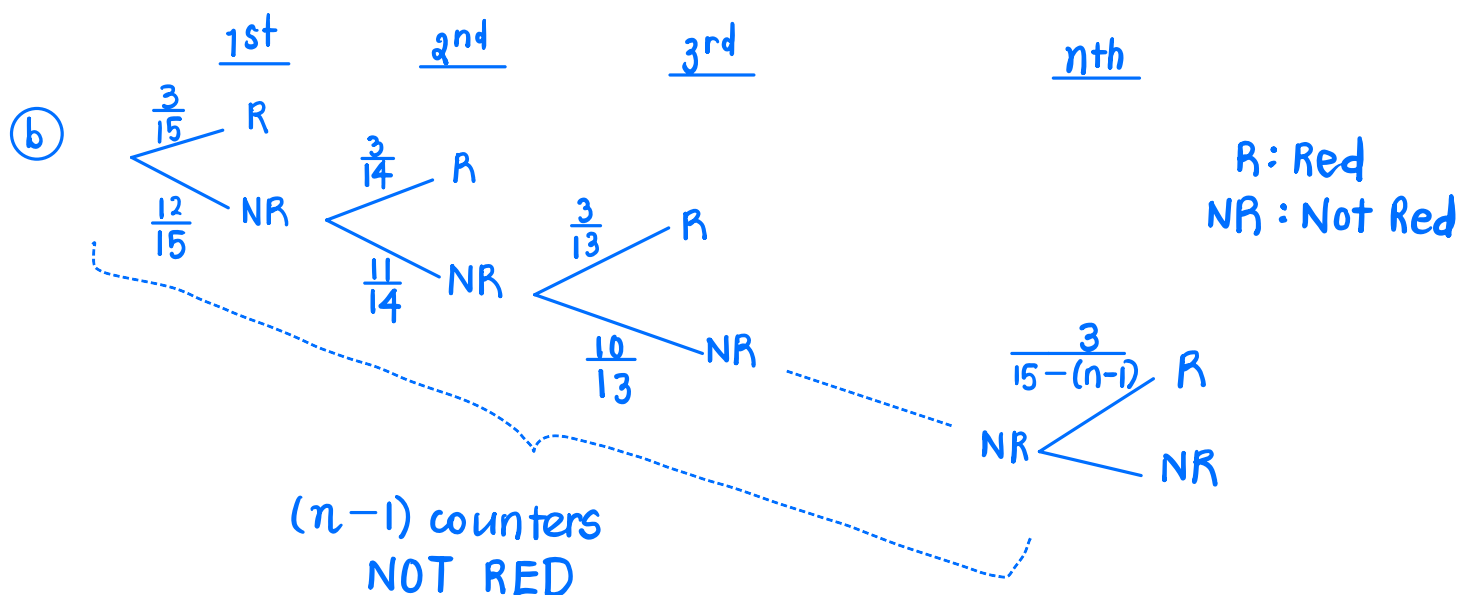
Section B: Probability and Statistics [60 marks]

- 5 Anya and Ben each have a bag containing 3 red counters, 5 blue counters and 7 yellow counters.
- (a) Anya takes 2 counters at random from her bag without replacement. Find the probability that the 2 counters are of different colours. [3]
- (b) Ben takes counters at random from his bag one at a time without replacement. The probability that his first red counter is the n th counter he takes is $\frac{36}{455}$. Find the value of n . [2]

① $\{R, B\} + \{B, Y\} + \{Y, R\}$

$$= \left(\frac{3}{15} \times \frac{5}{14} \times 2! \right) + \left(\frac{5}{15} \times \frac{7}{14} \times 2! \right) + \left(\frac{7}{15} \times \frac{3}{14} \times 2! \right)$$

$$= \frac{71}{105}$$



$P(\text{1st } n-1 \text{ counters are not red and } n\text{th counter is red})$

$$= \frac{{}^{12}C_{n-1}}{{}^{15}C_{n-1}} \times \frac{3}{15-(n-1)}$$

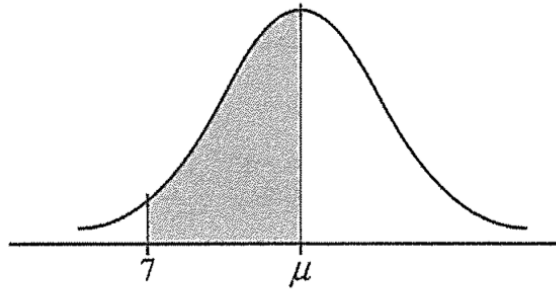
$$= \frac{{}^{12}C_{n-1}}{{}^{15}C_{n-1}} \times \frac{3}{16-n}$$

$$\text{Let } \frac{{}^{12}C_{n-1}}{{}^{15}C_{n-1}} \times \frac{3}{16-n} = \frac{36}{455} = 0.0791$$

By GC: $n = 6$

NORMAL FLOAT AUTO a+bi DEGREE MP PRESS [ENTER] TO EDIT					
X	Y1				
1	0.2				
2	0.1714				
3	0.1451				
4	0.1209				
5	0.0989				
6	0.0791				
7	0.0615				
8	0.0462				
9	0.033				
10	0.022				
11	0.0132				

$Y1 = \left(\frac{{}^{12}C_{X-1}}{{}^{15}C_{X-1}} \right) * \left(\frac{3}{16-n} \right)$



The random variable X is normally distributed with mean μ , as shown in the diagram. The shaded region between 7 and μ represents 45% of the distribution.

(a) Find $P(X < 7)$. [1]

The standard deviation of X is 1.8.

(b) Find the value of μ . [2]

The random variable Y has the distribution $N(\lambda, 2.8^2)$. The random variables X and Y are independent, and $P((X > 7) \cap (Y > 7)) = 0.38$.

(c) Find the value of λ . [3]

(d) Given that $P(Y > a) = 2P(Y > 7)$, find the value of a . [2]

$$\textcircled{a} P(X < 7) = 0.05$$

$$\textcircled{b} X \sim N(\mu, 1.8^2)$$

$$P(X < 7) = 0.05$$

$$P\left(Z < \frac{7 - \mu}{1.8}\right) = 0.05$$

$$\frac{7 - \mu}{1.8} = -1.64485$$

$$\mu = 7 + 1.64485(1.8)$$

$$= 9.9607$$

$$= 9.96$$

$\textcircled{c}$ Given X and Y are independent and $P(X > 7) \cap P(Y > 7) = 0.38$

$$\therefore P(X > 7) \cdot P(Y > 7) = 0.38$$

$$[1 - P(X \leq 7)] \cdot P(Y > 7) = 0.38$$

$$[1 - 0.05] \cdot P(Y > 7) = 0.38$$

$$P(Y > 7) = 0.4$$

$$P\left(Z > \frac{7 - \lambda}{2.8}\right) = 0.4$$

$$\frac{7 - \lambda}{2.8} = 0.253347$$

$$\lambda = 7 - 2.8(0.253347)$$

$$= 6.2906$$

$$= 6.29$$

$$\textcircled{d} P(Y > a) = 2 \cdot P(Y > 7)$$

$$= 2 \times 0.4$$

$$= 0.8$$

$$Y \sim N(6.2906, 2.8^2)$$

$$\text{By GC, } a = 3.93406$$

$$= 3.93$$



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7 Fred has 8 cards with the following digits on them.

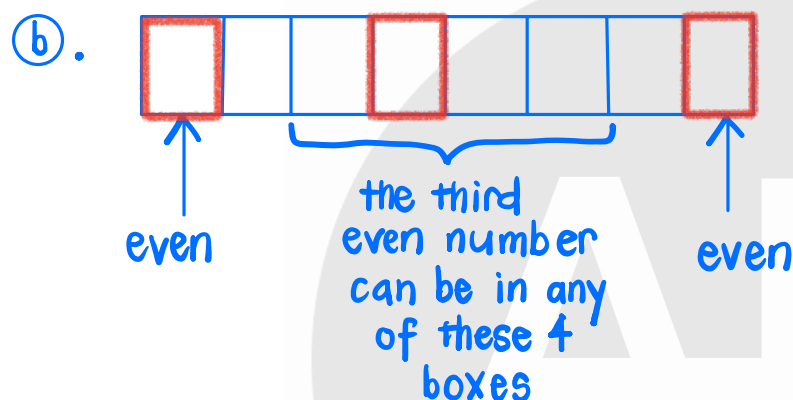
2 3 3 4 5 7 7 8

Fred places the 8 cards in a row.

- (a) Find the number of different arrangements that can be made. [2]
- (b) Find the number of different arrangements that can be made which start and end with an even digit and have no two even digits next to each other. [3]
- (c) Find the number of different arrangements that can be made in which the even digits are all together. [2]
- (d) Given that the even digits are all together, find the probability that the odd digits are also all together. [1]

(a) . $\frac{8!}{2! 2!}$
 $= 10080$

Because "3" and "7" are repeated



only "2", "4" and "8" are even numbers

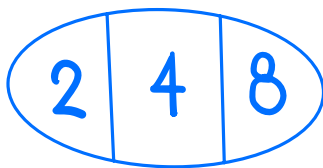
$3P_2 \times 4C_1 \times \frac{5!}{2!2!} = 720 \text{ ways}$

Select 2 even numbers from the available 3, and arrange them either at the front or at the back.

Choose 1 position out of the available 4, for the last remaining even number.

Arrange the 5 odd numbers in the remaining 5 boxes and don't forget that "3" and "7" are repeated.

©.



$$3! \times \frac{6!}{2! 2!} = 1080 \text{ ways}$$

Arrange "2",
"4" and "8"
in a packet

Arrange all
the 6 packets
in a linear
arrangement and
don't forget "3"
and "7" are repeated.

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$$\begin{aligned} \text{d) } & P(\text{odd digits are all together} \mid \text{even digits are all together}) \\ &= \frac{P(\text{odd digits are all together AND even digits are all together})}{P(\text{even digits are all together})} \end{aligned}$$

$$= \frac{\begin{array}{|c|c|c|} \hline 2 & 4 & 8 \\ \hline \end{array} \quad \begin{array}{|c|c|c|c|c|} \hline 3 & 3 & 5 & 7 & 7 \\ \hline \end{array}}{\left(\frac{1080}{10080} \right)}$$

$$= \frac{\left(3! \times \frac{5!}{2! 2!} \times 2! \right)}{\left(\frac{1080}{10080} \right)}$$

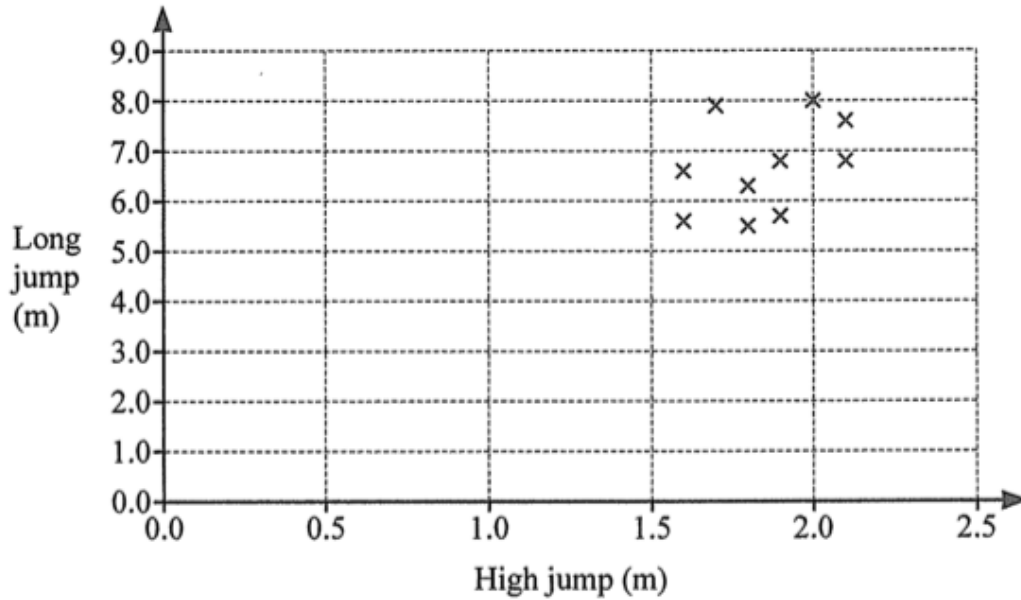
$$= \frac{360}{1080}$$

$$= \frac{1}{3}$$



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- 8 (a) Dilip is investigating how athletes perform in related events. He collects data about the best performances of 10 randomly chosen athletes in high jump and long jump. The results, in metres, are shown in the scatter diagram. The correlation coefficient is 0.39.



What do the scatter diagram and correlation coefficient suggest about the relationship between best performance in high jump and long jump amongst athletes taking part in both events? Justify your answer. [2]

Since $r = 0.39$, it suggests a weak, positive linear correlation between the best performance in high jump and long jump.

The scatter plot does not show a clear relationship between the 2 variables.



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- (b) Each evening, after work, Dilip goes for a run. The table below shows the distance he runs, x km, and the time he takes, t minutes, for a random sample of 8 runs.

x	2.1	3.9	6.2	7.8	10	12.5	13.8	15
t	11	26	53	75	119	165	246	300

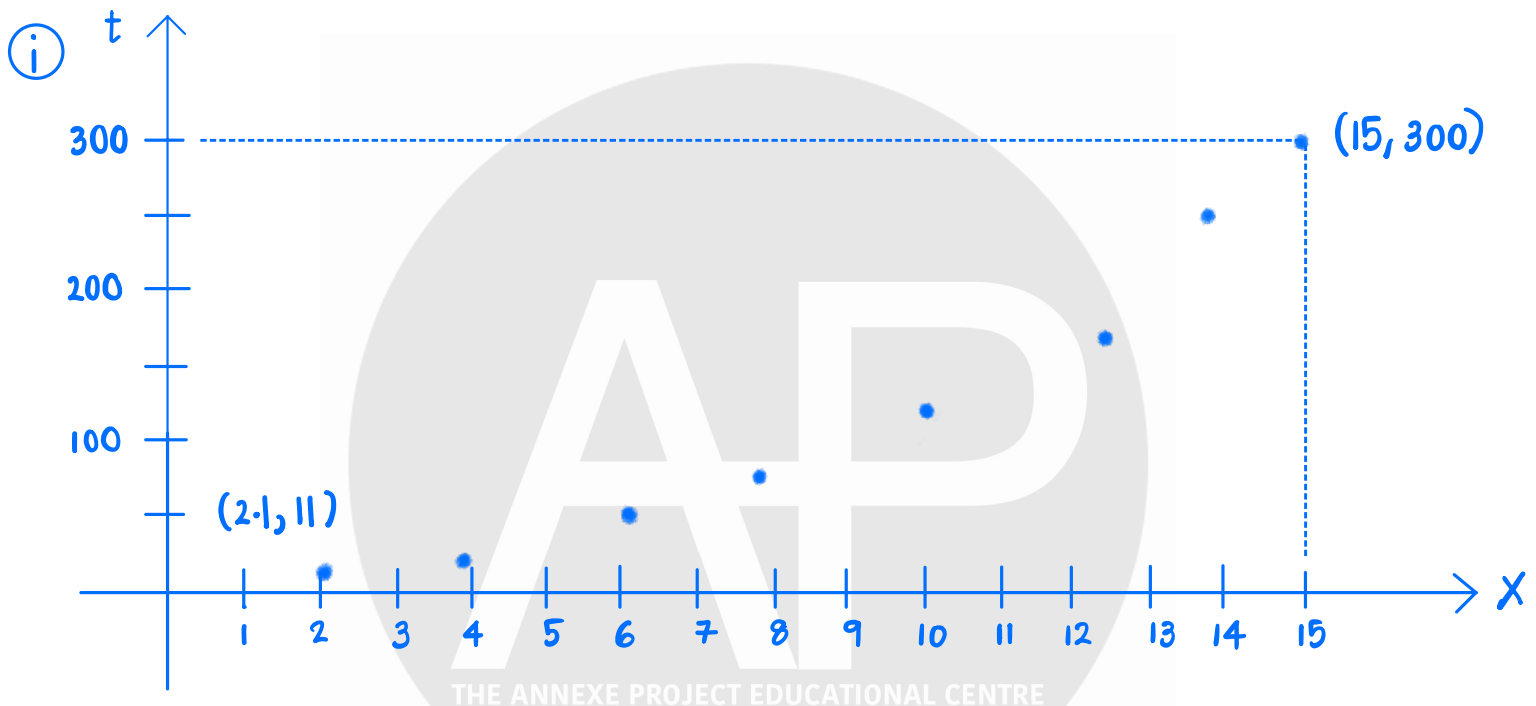
- (i) Sketch a scatter diagram of the data. State what the scatter diagram suggests about the relationship between x and t . [2]
- (ii) (A) Determine which of the following models is the better fit to the data where a , b , c and d are constants.

$$t = ax + b \quad t = cx^2 + d \quad [2]$$

- (B) Find the regression equation for the model identified in part (ii)(A). [2]

- (iii) (A) Use the regression equation found in part (ii)(B) to estimate the time Dilip would take for a run of 11 km. [1]

- (B) Give two reasons why your estimate is reliable. [2]



ii A. r between t and x : 0.958

r between t and x^2 : 0.990

since $|r|$ is closer to 1 for the model between t and x^2 , the model $t = cx^2 + d$ is a better fit.

B. from the GC : $t = 1.2497x^2 + 0.87581$

iii) When $x = 11$: $t = 152.09$ mins
 $= 152$ mins

- Interpolation was practised as $x = 11$ is within the range of data collected.
- $r = 0.990$ is close to 1, indicating strong linear correlation between t and x^2 .

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- 9 A company produces a variety of domestic appliances. One of Alan's jobs is to test a fixed number of randomly chosen refrigerators each day to see if any are faulty. The number found to be faulty each day is denoted by X .

(a) State, in context, two assumptions needed for X to be well-modelled by a binomial distribution. [2]

Assume now that X has the distribution $B(90, 0.02)$.

(b) Find the probability that, on a randomly chosen day, Alan finds more than one faulty refrigerator. [2]

(c) Find the probability that, in a randomly chosen 5-day working week, Alan finds more than one faulty refrigerator on fewer than two days. [2]

(d) Find the probability that, in a randomly chosen 5-day working week, Alan finds fewer than 10 faulty refrigerators in total. [2]

① • The event that a refrigerator is faulty or not is independent of another refrigerator.

• The probability that a refrigerator is faulty is constant.

②
$$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) \\ &= 0.53957 \\ &= 0.540 \end{aligned}$$

③ Let V be the r.v. denoting the number of days out of 5, that Alan finds more than 1 faulty refrigerator.

$$V \sim B(5, 0.53957)$$

$$\begin{aligned} P(V < 2) &= P(V \leq 1) = 0.14194 \\ &= 0.142 \end{aligned}$$

④ Let W be the r.v. denoting the number of faulty refrigerators out of 450.

$$W \sim B(450, 0.02)$$

$$\begin{aligned} P(W < 10) &= P(W \leq 9) = 0.58741 \\ &= 0.587 \end{aligned}$$

The company also makes washing machines. Alan also tests a fixed number of randomly chosen washing machines each day. The number of washing machines found to be faulty each day is denoted by Y . The distribution of Y is $B(60, 0.03)$. The random variables X and Y are independent.

- (e) Find the probability that, on a randomly chosen day, the total number of refrigerators and washing machines Alan finds to be faulty is exactly two. [3]

$$\begin{aligned}
 & P(X=2 \text{ and } Y=0) + P(X=1 \text{ and } Y=1) + P(X=0 \text{ and } Y=2) \\
 &= P(X=2) \cdot P(Y=0) + P(X=1) \cdot P(Y=1) + P(X=0) \cdot P(Y=2) \\
 &= (0.27074) \cdot (0.16081) + (0.29812) \cdot (0.29840) + (0.16231) \cdot (0.27226) \\
 &= 0.17669 \\
 &= 0.177
 \end{aligned}$$



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10 In a food processing plant, two different machines are used to produce packets of flour. Over a long period of time it has been established that the masses, in kg, of packets produced by Machine P follow the distribution $N(2.2, 0.1^2)$ and the masses, in kg, of packets produced by Machine Q follow the distribution $N(2.1, 0.05^2)$. It is assumed that these two distributions are independent.

- (a) Explain why, in the context of the question, it is reasonable to assume that the two distributions are independent. [1]
- (b) Find the probability that a randomly chosen packet produced by Machine P has a greater mass than a randomly chosen packet produced by Machine Q . [3]
- (c) Find the probability that 3 randomly chosen packets produced by Machine P and 5 randomly chosen packets produced by Machine Q have a total mass greater than 17 kg. [3]

(a) P and Q are two separate machines, hence they should not affect each other's production of flour.

(b) $P \sim N(2.2, 0.1^2)$

$Q \sim N(2.1, 0.05^2)$

$P - Q \sim N(0.1, 0.0125)$

$P(P > Q) = P(P - Q > 0)$

$= 0.81445$

$= 0.814$

(c) $E(P_1 + P_2 + P_3 + Q_1 + Q_2 + Q_3 + Q_4 + Q_5)$

$= (3 \times 2.2) + (5 \times 2.1)$

$= 17.1$

$\text{Var}(P_1 + P_2 + P_3 + Q_1 + Q_2 + Q_3 + Q_4 + Q_5)$

$= (3 \times 0.1^2) + (5 \times 0.05^2)$

$= 0.0425$

$\therefore P_1 + P_2 + P_3 + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 \sim N(17.1, 0.0425)$

$P(P_1 + P_2 + P_3 + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 > 17) = 0.68619$
 $= 0.686$

Following an adjustment to Machine P , the production manager wishes to test if the mean mass of packets produced by that machine now differs from 2.2 kg.

- (d) State hypotheses for the production manager's test, defining any parameter that you use. [2]

The production manager finds the masses of a random sample of 30 packets produced by Machine P . The masses, x kg, are summarised below.

$$\sum(x-2) = 4.5 \quad \sum(x-2)^2 = 1.11$$

- (e) Calculate unbiased estimates of the population mean and variance of the masses of packets after Machine P has been adjusted. [2]
- (f) Carry out the production manager's test at the 5% level of significance. Show the values you use to carry out the test and give your conclusion in the context of the question. [4]

- ④ Let μ be the population mean mass of a packet of flour produced by machine P .

Test $H_0 : \mu = 2.2 \text{ kg}$ against $H_1 : \mu \neq 2.2 \text{ kg}$

where $H_0 = \text{null hypothesis}$

$H_1 = \text{alternate hypothesis}$

⑤ $\bar{x} = \frac{\sum(x-2)}{30} + 2 = \frac{4.5}{30} + 2 = 2.15 \text{ kg}$

$$s^2 = \frac{1}{n-1} \left[\sum(x-2)^2 - \frac{[\sum(x-2)]^2}{n} \right]$$

$$= \frac{1}{29} \left[1.11 - \frac{4.5^2}{30} \right] = 0.015 \text{ kg}^2$$

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- ⑥ Step 1: To test $H_0 : \mu = 2.2 \text{ kg}$ against $H_1 : \mu \neq 2.2 \text{ kg}$ at 5% level of sig.

Step 2: Under H_0 , and since $n = 30$ is large by CLT, $\bar{X} \sim N(2.2, \frac{0.015}{30})$ approx.

Test statistic: $Z = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \sim N(0, 1)$

Step 3: By GC, $p\text{-value} = 0.025347 < 0.05$

Step 4: Since $p < 0.05$, we reject H_0 at 5% level of significance. There is sufficient evidence to conclude that the population mean mass of packets produced by machine P differs from 2.2 kg after the adjustment.

(g) Explain why the production manager would have less confidence in the conclusion of the hypothesis test if a sample of fewer than 30 packets had been used. [1]

(h) Explain why the production manager would have less confidence in the conclusion of the hypothesis test if the sample had not been chosen randomly. [1]

⑨ If sample size is fewer than 30, we cannot apply Central Limit theorem. And Z-test may not be applicable for the hypothesis test.

⑩ The sample could be biased.



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