MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level

MATHEMATICS **9758/01**

Paper 1 **October/November 2026**

3 hours

You must answer on the printed answer booklet.

You will need: Printed answer booklet
List of formulae and results (MF27)

INSTRUCTIONS

- Answer **all** questions.
- Write your answers on the printed answer booklet. Follow the instructions on the front cover of the answer booklet.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- You should use an approved graphing calculator.
- Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
- Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
- You must show all working clearly.

INFORMATION

- The total mark for this paper is 100.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Any blank pages are indicated.

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[Turn over

1 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_{n+1} = Au_n + Bn + C$, where $n \geqslant 1$ and A, B and C are constants. It is given that $u_1 = 8$, $u_2 = 22$, $u_3 = 49$ and $u_4 = 102$.

Find the value of u_5. [4]

Given $U_{n+1} = AU_n + Bn + C$

Let $n=1$: $U_2 = AU_1 + B(1) + C$

$22 = 8A + B + C$ —— (1)

Let $n=2$: $U_3 = AU_2 + B(2) + C$

$49 = 22A + 2B + C$ —— (2)

Let $n=3$: $U_4 = AU_3 + B(3) + C$

$102 = 49A + 3B + C$ —— (3)

By GC: $A = 2$, $B = -1$, $C = 7$

$\therefore U_{n+1} = 2U_n - n + 7$

$U_5 = 2U_4 - 4 + 7$

$= 2(102) - 4 + 7$

$= 207$

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2 An athlete is running in an ultramarathon race. The distance travelled by the athlete is modelled by the following.

- In the first 15 minutes she runs a distance of 2.5 km.
- In subsequent 15-minute time intervals she runs 95% of the distance she ran in the previous 15-minute time interval.

(a) According to the model, find the distance she runs between 1 hour and 1 hour 15 minutes after the start. [1]

The total distance of this ultramarathon is 52 km.

(b) According to the model, find the 15-minute time interval in which the athlete will reach the half-way point. [3]

(c) According to the model, determine whether she will complete the ultramarathon, and hence explain whether the model is suitable. [3]

(a) Between 1 h to 1 h 15 mins = 5th interval

distance $= (0.95)^4 (2.5)$

$= 2.0363$

$= \underline{2.04 \text{ Km}}$

(b) Half way point = 26 km

$S_n = \frac{a(1-r^n)}{1-r} = \frac{2.5(1-0.95^n)}{1-0.95}$

$= 50(1-0.95^n)$

Let $50(1-0.95^n) = 26$

$1-0.95^n = \frac{26}{50}$

$0.95^n = \frac{24}{50}$

$n \lg 0.95 = \lg \frac{12}{25}$

$n = 14.309$

Check: Let $n = 14$ (i.e. 14th interval)

$S_{14} = 50(1-0.95^{14})$

$= 25.6$ km

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Let $n = 15$ (i.e. 15^{th} interval)

$$S_{15} = 50(1 - 0.95^{15})$$
$$= 26.8 \text{ km}$$

The athlete will reach the half-way point on the 15^{th} 15-minute time interval, i.e. 3 h 30 min to 3 h 45 min.

(c) $$S_\infty = \frac{a}{1-r}$$
$$= \frac{2.5}{1-0.95}$$
$$= 50 \text{ km}$$

According to this model, the total distance she could cover will always be less than 50 km. Hence, she will not be able to complete the ultramarathon.

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3 Relative to the origin O, the distinct points A, B, C and D have position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ respectively.

(a) It is given that $(\mathbf{a}-\mathbf{b})\times(\mathbf{a}-\mathbf{c})=\mathbf{0}$. State the relationship between the points A, B and C, justifying your answer. [2]

It is given that $\mathbf{b}$ is a unit vector.

(b) Give a geometrical interpretation of $|\mathbf{b}.(\mathbf{c}-\mathbf{d})|$. [1]

It is also given that $\mathbf{a}=\begin{pmatrix}2\\-3\\4\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}p\\q\\q\end{pmatrix}$, where p and q are constants, and that OA is perpendicular to OB.

(c) Find the possible values of p and q. [4]

(a) $(\underset{\sim}{a}-\underset{\sim}{b})\times(\underset{\sim}{a}-\underset{\sim}{c})=0$

$\overrightarrow{BA}\times\overrightarrow{CA}=0$

hence $\overrightarrow{BA}$ is parallel to $\overrightarrow{CA}$.

Since A is the common point, A, B and C are collinear points.

(b) $|\underset{\sim}{b}\cdot(\underset{\sim}{c}-\underset{\sim}{d})|=|(\underset{\sim}{c}-\underset{\sim}{d})\cdot\underset{\sim}{b}|$

$=|\overrightarrow{DC}\cdot\hat{\underset{\sim}{b}}|$ (since $\underset{\sim}{b}=\hat{\underset{\sim}{b}}$)

= length of projection of $\overrightarrow{DC}$ on $\underset{\sim}{b}$.

(c) $\underset{\sim}{a}\cdot\underset{\sim}{b}=0$

$\begin{pmatrix}2\\-3\\4\end{pmatrix}\cdot\begin{pmatrix}p\\q\\q\end{pmatrix}=0$

$2p-3q+4q=0$

$2p+q=0$

$q=-2p$ —(1)

$\underset{\sim}{b}$ is a unit vector

$\therefore\sqrt{p^2+q^2+q^2}=1$

$p^2+2q^2=1$ —(2)

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Sub ① into ②: $p^2 + 2(-2p)^2 = 1$

$p^2 + 8p^2 = 1$

$9p^2 = 1$

$p = \pm \frac{1}{3}$

When $p = -\frac{1}{3}$, $q = \frac{2}{3}$

When $p = \frac{1}{3}$, $q = -\frac{2}{3}$

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4 (a) Use the substitution $y = vx$ to show that the differential equation

$$x^2\frac{dy}{dx} = y^2 + xy$$

reduces to

$$x\frac{dv}{dx} = v^2.$$

[3]

(b) Hence find the general solution of the differential equation

$$x^2\frac{dy}{dx} = y^2 + xy.$$

Give your answer in the form $y = f(x)$, where $f(x)$ is a function to be determined. [3]

(a) Given $y = vx$

differentiate both sides w.r.t. x

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{——①}$$

Sub ① into $x^2\frac{dy}{dx} = y^2 + xy$

$$x^2\left(v + x\frac{dv}{dx}\right) = (vx)^2 + x(vx)$$

$$vx^2 + x^3\frac{dv}{dx} = v^2x^2 + vx^2$$

$$x^3\frac{dv}{dx} = x^2v^2$$

divide both sides by x^2:

$$x\frac{dv}{dx} = v^2 \quad \text{(shown)}$$

(b) $\int v^{-2}\,dv = \int \frac{1}{x}\,dx$

$$\frac{v^{-1}}{-1} = \ln|x| + c$$

$$-\frac{1}{v} = \ln|x| + c$$

$$-\frac{1}{\left(\frac{y}{x}\right)} = \ln|x| + c$$

$$-\frac{x}{y} = \ln|x| + c$$

$$\frac{y}{-x} = \frac{1}{\ln|x| + c}$$

$$y = \frac{-x}{\ln|x| + c}, \text{ where } c \text{ is an arbitrary constant.}$$

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5 **Do not use a calculator in answering this question.**

Find the complex numbers z and w which satisfy these simultaneous equations.

$$2z+7i=-iw$$
$$z-4=w-4i$$

Give your answers in the form $a+ib$, where a and b are real numbers. [5]

$2z + 7i = -iw$ —①

$z - 4 = w - 4i$

$z = w + 4 - 4i$ —②

Sub ② into ①:

$2(w + 4 - 4i) + 7i = -iw$

$2w + 8 - 8i + 7i = -iw$

$2w + iw = -8 + i$

$w(2 + i) = -8 + i$

$$w = \frac{-8+i}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{-15 + 10i}{4 + 1}$$

$$= -3 + 2i$$

$\therefore z = (-3 + 2i) + 4 - 4i$

$= 1 - 2i$

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6 (a) Solve the inequality $\frac{2}{x+1} < 3-2x$. [4]

(b) Hence solve exactly the inequality $\frac{2}{\sin\theta+1} < 3-2\sin\theta$, for $0 < \theta < 2\pi$. [3]

(a) $\frac{2}{x+1} < 3-2x$

$\frac{2}{x+1} - 3 + 2x < 0$

$\frac{2+(2x-3)(x+1)}{x+1} < 0$

$\frac{2x^2-x-1}{x+1} < 0$

$\frac{(2x+1)(x-1)}{(x+1)} < 0 \quad , \quad x \neq -1$



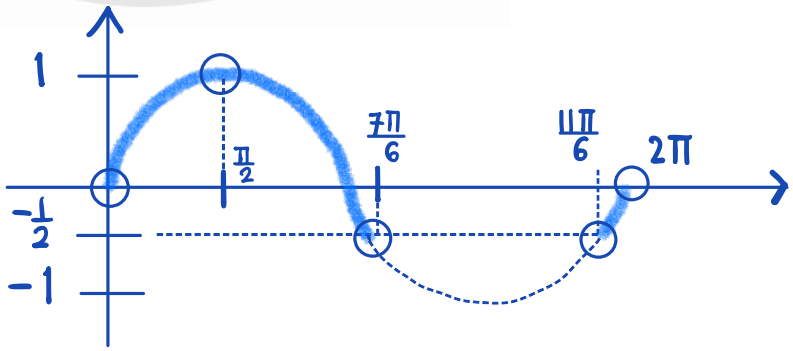
$x < -1$ or $-\frac{1}{2} < x < 1$

(b) Let $x = \sin\theta$:

$\sin\theta < -1$

(No Solutions)

$-\frac{1}{2} < \sin\theta < 1$



from graphs, $0 < \theta < \frac{\pi}{2}$, $\frac{\pi}{2} < \theta < \frac{7\pi}{6}$ or $\frac{11\pi}{6} < \theta < 2\pi$

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7 (a) Differentiate $e^{\cos 2x}$ with respect to x. [2]

(b) Hence, find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} e^{\cos 2x} \sin 4x \, dx$. [5]

ⓐ $\frac{d}{dx} e^{\cos 2x} = e^{\cos 2x} \cdot (-2 \sin 2x)$

ⓑ $\int e^{\cos 2x} \sin 4x \, dx$

$= \int e^{\cos 2x} (2 \sin 2x \cos 2x) \, dx$

$= \int \underbrace{(-\cos 2x)}_{u} \underbrace{(-2 e^{\cos 2x} \sin 2x)}_{dv} dx$

$= (-\cos 2x)(e^{\cos 2x}) - \int e^{\cos 2x} \cdot 2 \sin 2x \, dx$

$= -\cos 2x \, e^{\cos 2x} + \int e^{\cos 2x} (-2 \sin 2x) \, dx$

$= -\cos 2x \, e^{\cos 2x} + e^{\cos 2x} + C$

Hence, $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\cos 2x} \sin 4x \, dx$

$= \left[-\cos 2x \, e^{\cos 2x} + e^{\cos 2x} \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= [(1)e^{-1} + e^{-1}] - [0 + e^{0}]$

$= \frac{2}{e} - 1$

Double Angles Formula

Since $\sin 2x = 2 \sin x \cos x$

$\therefore \sin 4x = 2 \sin 2x \cos 2x$

Using by Parts

Let $u = -\cos 2x$

$\frac{du}{dx} = 2 \sin 2x$

Let $dv = -2 e^{\cos 2x} \sin 2x$

$v = e^{\cos 2x}$ ——— from ⓐ

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8 (a) A curve C has equation $y = \frac{2x^2}{2x-1}$.

Find the set of values that y can not take. [3]

(b) A curve D has equation $y = \frac{4x^2+1}{2x-1}$.

(i) Express $y = \frac{4x^2+1}{2x-1}$ in the form $y = ax+b+\frac{c}{2x-1}$, where a, b and c are constants.

Hence write down the equations of the asymptotes of D. [3]

(ii) Describe a sequence of two transformations that transforms the graph of $y = x+\frac{1}{2x-1}$ onto D. [2]

(a) $y = \frac{2x^2}{2x-1}$

$2yx - y = 2x^2$

$2x^2 - 2yx + y = 0$

$b^2 - 4ac < 0$

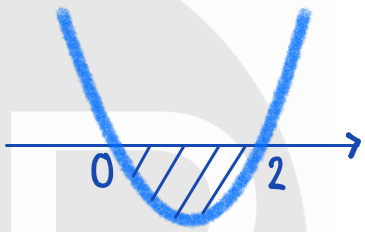
$(-2y)^2 - 4(2)(y) < 0$

$4y^2 - 8y < 0$

$y^2 - 2y < 0$

$y(y-2) < 0$

$0 < y < 2$



∴ set of values y cannot take $\{y : y \in \mathbb{R}, 0 < y < 2\}$

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(b) i. Using Long Division,

$$2x-1 \overline{)\,4x^2+0x+1}$$

$2x+1$

$-(4x^2-2x)$

$2x+1$

$-(2x-1)$

2

∴ $\frac{4x^2+1}{2x-1} = 2x+1+\frac{2}{2x-1}$

Vertical asymptote: $x = \frac{1}{2}$

Oblique asymptote: $y = 2x+1$

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ii.

$y = x + \frac{1}{2x-1}$ $\xrightarrow[A]{\text{replace } y \text{ with } \frac{y}{2}}$ $\frac{y}{2} = x + \frac{1}{2x-1}$

$\therefore y = 2x + \frac{2}{2x-1}$ $\xrightarrow[B]{\text{replace } y \text{ with } y-1}$ $y - 1 = 2x + \frac{2}{2x-1}$

$\therefore y = 2x + 1 + \frac{2}{2x-1}$

Transformation A : scaling of factor 2 parallel to y-axis.

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B : translation of 1 unit along positive y-axis.

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9 It is given that $f(x) = \ln(a + a\sin x)$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, where a is a positive constant.

(a) Show that $f''(x) = \frac{k}{1+\sin x}$, where k is a constant to be found. [3]

(b) By further differentiation of the result in **9(a)**, find the Maclaurin series for $f(x)$, up to and including the term in x^3. [4]

(c) It is now given that $a = 2$.

(i) Use the series found in **9(b)** to find an approximate value of $\int_0^{1.2} f(x)\,dx$, giving your answer to 2 decimal places. [2]

(ii) Use a calculator to evaluate $\int_0^{1.2} f(x)\,dx$, giving your answer to 2 decimal places. [1]

(iii) Explain why your answers to **9(c)(i)** and **9(c)(ii)** are different. [1]

(a) $f(x) = \ln(a + a\sin x)$

$$f'(x) = \frac{1}{a + a\sin x} \cdot a\cos x = \frac{a\cos x}{a + a\sin x}$$

$$= \frac{\cos x}{1 + \sin x}$$

$$f''(x) = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1}{(1+\sin x)^2}$$

$$= \frac{-(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{-1}{1+\sin x} \quad \text{where } \underline{k = -1}$$

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(b) $f''(x) = -(1+\sin x)^{-1}$

$\therefore f'''(x) = (1+\sin x)^{-2}(\cos x) = \frac{\cos x}{(1+\sin x)^2}$

When $x = 0$,

$f(0) = \ln a$, $f'(0) = 1$, $f''(0) = -1$, $f'''(0) = 1$

hence, $f(x) = \ln a + x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots$

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© i. $\int_0^{1.2} \ln(2+2\sin x)\,dx$

$\approx \int_0^{1.2} \ln 2 + x - \frac{1}{2}x^2 + \frac{1}{6}x^3\,dx$

$= \left[x\ln 2 + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}\right]_0^{1.2}$

$= 1.350176617$

$= 1.35$

ii. $\int_0^{1.2} \ln(2+2\sin x)\,dx = 1.322161427$

$= 1.32$ (by GC)

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iii. For the approximation in © i to be closer to the true value, x should be small. $x = 1.2$ is not small enough for x^4 and above to be negligible.

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10 The function $f(x)$ is defined by $f: x \mapsto e^{|3-x|}$, $x \in \mathbb{R}$, $x \leqslant m$.

(a) Explain why 3 is the greatest value of m for which the inverse function f^{-1} exists. [1]

For the remainder of the question, $m = 3$.

(b) Find $f^{-1}(x)$ and state its domain. [3]

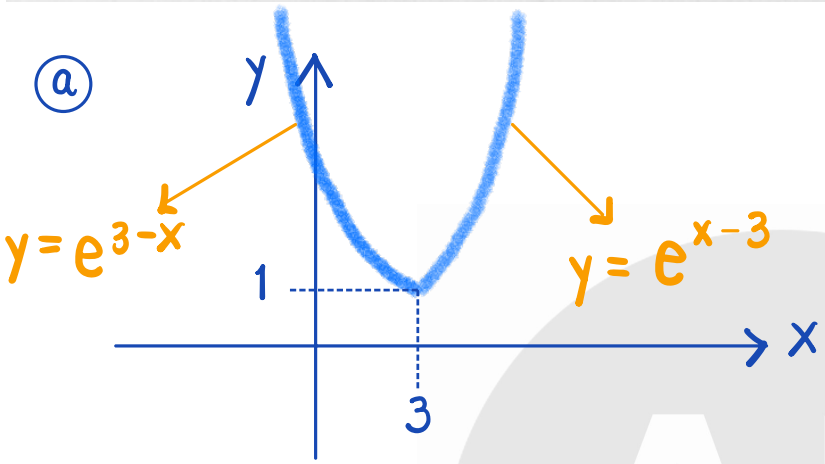
(c) Solve the inequality $f(f^{-1}(|x|)) \geqslant 1$. [3]

The function $g(x)$ is defined by $g: x \mapsto n + \sqrt{x-4}$, $x \in \mathbb{R}$, $x \geqslant 4$ and $n \in \mathbb{R}$.

(d) Find the smallest value of n for the composite function $f^{-1}g(x)$ to exist.

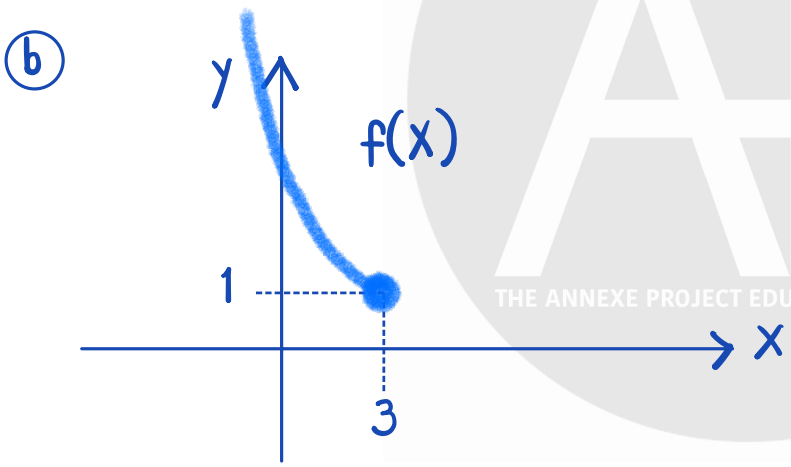
Justify your answer. [2]

(a)



because $x = 3$ is the turning point of $f(x)$.

(b)



$D_f = (-\infty, 3]$

$R_f = [1, \infty)$

$y = e^{3-x}$

$\ln y = 3 - x$

$x = 3 - \ln y$

$\therefore f^{-1}(x) = 3 - \ln x \quad , x \geqslant 1$

$D_{f^{-1}} = R_f = [1, \infty)$

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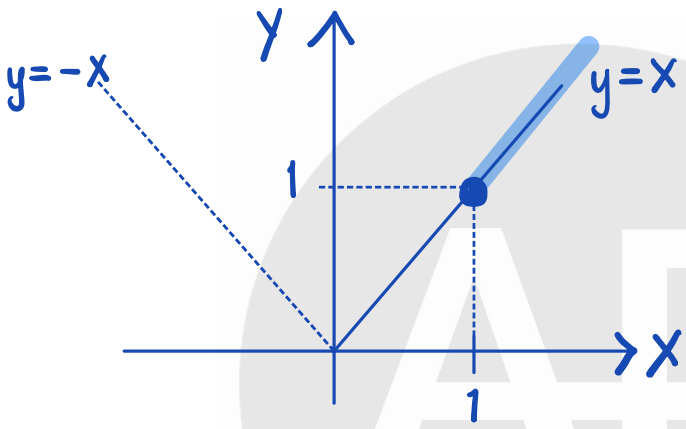
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(c) $D_{ff^{-1}} = D_{f^{-1}} = [1, \infty)$

Given $f(f^{-1}(|x|)) \geq 1$

$$|x| \geq 1$$

$$\therefore x \leq -1 \quad \text{or} \quad x \geq 1$$



But, because $D_{ff^{-1}} = [1, \infty)$, $x \geq 1$.

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(d) For $f^{-1}g(x)$ to exist, $R_g \subseteq D_{f^{-1}}$,

i.e. $R_g \subseteq [1, \infty)$

Given $g(x) = n + \sqrt{x-4}$, $x \geq 4$

when $x = 4$, $g(x) \geq 1$

$$n + \sqrt{4-4} \geq 1$$

$$n \geq 1$$

$\therefore$ least value of $n = 1$

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11 The parametric equations of a curve C are

$$x = 2\theta + \sin 2\theta, \quad y = 2\sin\theta \text{ for } 0 \leqslant \theta \leqslant 2\pi.$$

The graph of C meets the x-axis at the origin and the point Q. The graph of C crosses the x-axis at the point P.

(a) Sketch the graph of C and determine the exact coordinates of P and Q. [4]

(b) (i) Show that $\frac{dy}{dx} = \frac{1}{2}\sec\theta$. [4]

(ii) Hence explain why the graph of C has no stationary points. [1]

(iii) The tangent to C at the point where $\theta = \frac{1}{3}\pi$ cuts the x-axis and the y-axis at the points R and S respectively. Find the distance RS. [5]

(a) Let $y = 0$

$2\sin\theta = 0$

$\sin\theta = 0$

$\therefore \theta = 0, \pi$ or 2π

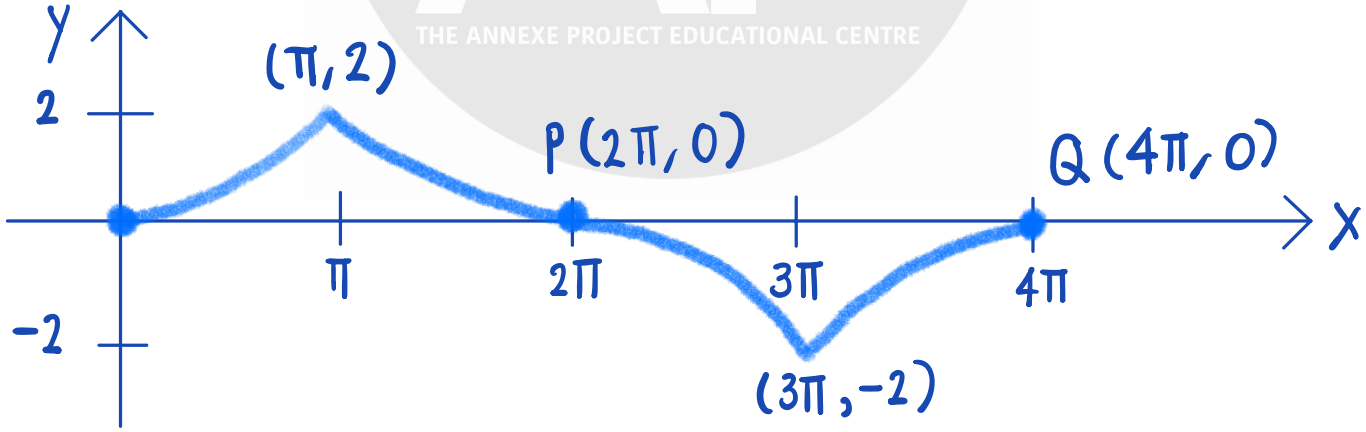
When $\theta = 0$, $x = 0$. $(0, 0)$

When $\theta = \pi$, $x = 2\pi + \sin 2\pi$

$= 2\pi$. $(2\pi, 0)$

When $\theta = 2\pi$, $x = 4\pi + \sin 4\pi$

$= 4\pi$. $(4\pi, 0)$



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(b) i. $x = 2\theta + \sin 2\theta$ $\quad y = 2\sin\theta$

$\frac{dx}{d\theta} = 2 + 2\cos 2\theta$ $\quad \frac{dy}{d\theta} = 2\cos\theta$

$\therefore \frac{dy}{dx} = \frac{2\cos\theta}{2 + 2\cos 2\theta}$

$= \frac{\cancel{2}\cos\theta}{\cancel{2}(1+\cos 2\theta)}$

$= \frac{\cos\theta}{\cancel{1} + 2\cos^2\theta \cancel{-1}}$

$= \frac{1}{2\cos\theta}$

$= \frac{1}{2}\sec\theta$ (shown)

ii. In order for the graph C to have stationary points, $\frac{dy}{dx}$ must be 0.
since $\frac{dy}{dx} = \frac{1}{2\cos\theta}$, the numerator of this expression is always 1 (i.e. not possible to be 0), hence $\frac{dy}{dx}$ will never be 0.

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iii. When $\theta = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{1}{2\cos\frac{\pi}{3}} = 1$

$x = 2\left(\frac{\pi}{3}\right) + \sin 2\left(\frac{\pi}{3}\right)$

$= \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

$y = 2\sin\frac{\pi}{3}$

$= 2\left(\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3}$

$\therefore$ equation of tangent: $y - \sqrt{3} = 1\left(x - \frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$y = x + \frac{\sqrt{3}}{2} - \frac{2\pi}{3}$

S: Let $x = 0$, $y = \frac{\sqrt{3}}{2} - \frac{2\pi}{3}$

R: Let $y = 0$, $x = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

distance $RS = \sqrt{\left(\frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right)^2 + \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)^2}$

$= \underline{1.74 \text{ units}}$

12 A mining company has identified a mineral layer below ground. The mining company intends to drill down to the mineral layer.

The ground is modelled as a plane with equation $z = 0$, with respect to an origin O.

The mineral layer is modelled as part of a plane that contains the points $A(10, 15, -60)$, $B(5, 8, -40)$ and $C(-5, 10, -50)$ where the units are metres.

(a) Find a cartesian equation of the plane containing A, B and C. [5]

(b) Find the acute angle between the ground and the mineral layer. [2]

ⓐ given ground as π_1: $\underset{\sim}{r} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$
(x-y plane)

given also $\overrightarrow{OA} = \begin{pmatrix} 10 \\ 15 \\ -60 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 8 \\ -40 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} -5 \\ 10 \\ -50 \end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ 8 \\ -40 \end{pmatrix} - \begin{pmatrix} 10 \\ 15 \\ -60 \end{pmatrix} = \begin{pmatrix} -5 \\ -7 \\ 20 \end{pmatrix}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} -5 \\ 10 \\ -50 \end{pmatrix} - \begin{pmatrix} 10 \\ 15 \\ -60 \end{pmatrix} = \begin{pmatrix} -15 \\ -5 \\ 10 \end{pmatrix} = 5\begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$

$\underset{\sim}{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} -5 \\ -7 \\ 20 \end{pmatrix} \times \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -50 \\ -16 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -25 \\ -8 \end{pmatrix}$

equation of mineral layer π_2: $\underset{\sim}{r} \cdot \begin{pmatrix} 3 \\ -25 \\ -8 \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \\ -60 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -25 \\ -8 \end{pmatrix}$

$= 30 - 375 + 480$

$= 135$

cartesian equation of π_2: $3x - 25y - 8z = 135$

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ⓑ $\theta = \cos^{-1} \left| \dfrac{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -25 \\ -8 \end{pmatrix}}{\sqrt{1}\ \sqrt{3^2 + 25^2 + 8^2}} \right|$

$= \cos^{-1} \left| \dfrac{-8}{\sqrt{698}} \right|$

$= 72.374$

$= 72.4°$

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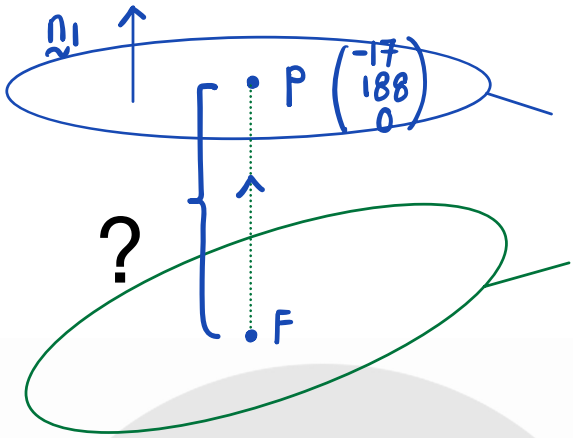
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The mining company plans to drill vertically from the point $P(-17, 188, 0)$ on the ground to reach the mineral layer.

(c) Find the vertical distance from P to the plane containing the mineral layer. [2]

(d) Find the shortest distance from P to the plane containing the mineral layer. [6]

(c)



π_1 (ground)

π_2 (mineral layer)

$\pi_2 : \underset{\sim}{r} \cdot \begin{pmatrix}3\\-25\\-8\end{pmatrix} = 135$

Step 1: $l_{\overrightarrow{PF}} : \underset{\sim}{r} = \begin{pmatrix}-17\\188\\0\end{pmatrix} + \lambda \begin{pmatrix}0\\0\\1\end{pmatrix}, \lambda \in \mathbb{R}$

Step 2: $\overrightarrow{OF} = \begin{pmatrix}-17\\188\\\lambda\end{pmatrix}$

$$\begin{pmatrix}-17\\188\\\lambda\end{pmatrix} \cdot \begin{pmatrix}3\\-25\\-8\end{pmatrix} = 135$$

$$-51 - 4700 - 8\lambda = 135$$

$$\lambda = -610.75$$

$$\therefore \overrightarrow{OF} = \begin{pmatrix}-17\\188\\-610.75\end{pmatrix}$$

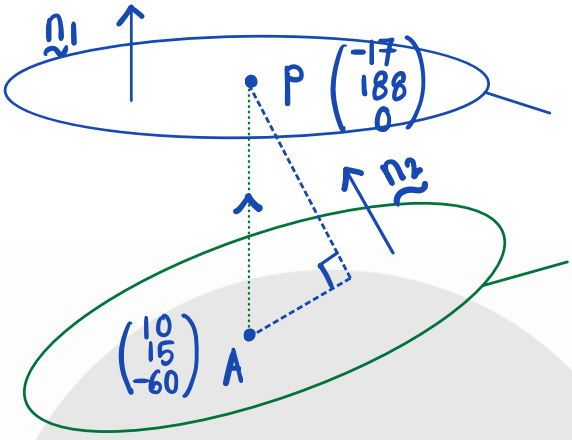
Hence F is 610.75 m vertically below P.

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(d)



$\pi_2 : \underset{\sim}{r} \cdot \begin{pmatrix}3\\-25\\-8\end{pmatrix} = 135$

Choose any point on π_2, e.g. $\overrightarrow{OA}$

Step 1: $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

$= \begin{pmatrix}-17\\188\\0\end{pmatrix} - \begin{pmatrix}10\\15\\-60\end{pmatrix} = \begin{pmatrix}-27\\173\\60\end{pmatrix}$

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Step 2: Shortest distance from P to π_2 (i.e. perpendicular dist.)

$= \left| \overrightarrow{AP} \cdot \hat{\underset{\sim}{n}} \right|$ (length of projection formula)

$= \dfrac{\left| \begin{pmatrix}-27\\173\\60\end{pmatrix} \cdot \begin{pmatrix}3\\-25\\-8\end{pmatrix} \right|}{\sqrt{3^2 + (-25)^2 + (-8)^2}}$

$= \dfrac{4886}{\sqrt{698}}$

$= 184.94$

$= \underline{185 \text{ m}}$

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