H2

MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level

MATHEMATICS **9758/02**

Paper 2 **October/November 2026**

3 hours

You must answer on the printed answer booklet.

You will need: Printed answer booklet
List of formulae and results (MF27)

INSTRUCTIONS

- Answer **all** questions.
- Write your answers on the printed answer booklet. Follow the instructions on the front cover of the answer booklet.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- You should use an approved graphing calculator.
- Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
- Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
- You must show all working clearly.

INFORMATION

- The total mark for this paper is 100.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages.

Singapore Examinations and Assessment Board

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[Turn over

Section A: Pure Mathematics [40 marks]

1 Find a unit vector that is perpendicular to both $3\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}$ and $2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. [4]

$$\underset{\sim}{n} = \begin{pmatrix} 3 \\ -2 \\ 10 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -26 \\ 26 \\ 13 \end{pmatrix}$$

$$= 13 \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

$$\hat{\underset{\sim}{n}} = \frac{\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}}{\sqrt{(-2)^2 + 2^2 + 1^2}} = \frac{\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}}{3} = \underline{\begin{pmatrix} -2/3 \\ 2/3 \\ 1/3 \end{pmatrix}}$$

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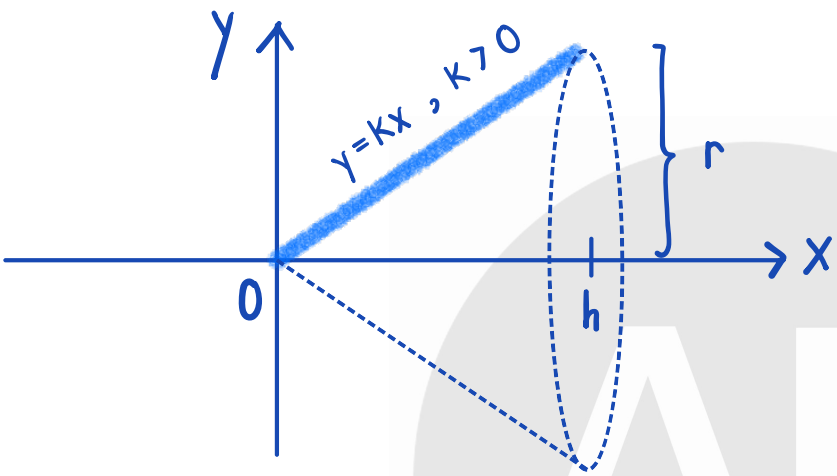
2 (a) (i) Use calculus to find the volume generated when the line $y = kx$, for $0 \leq x \leq h$, is rotated through 2π radians about the x −axis. [3]

(ii) Hence show that the volume of a cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$. [1]

@ i.



Using calculus,

$$V = \pi \int_0^h (kx)^2 \, dx$$
$$= \pi k^2 \left[\frac{x^3}{3} \right]_0^h$$
$$= \pi k^2 \left(\frac{h^3}{3} \right)$$
$$= \frac{\pi}{3} k^2 h^3 \text{ units}^3$$

Do not use formula for volume of cone: $V = \frac{1}{3}\pi r^2 h$

ii. from @ i,

$$V = \frac{\pi}{3} k^2 h^3 = \frac{1}{3}\pi (kh)^2 h$$

from the sketch above, radius of the cone = kh

hence, $V = \frac{1}{3}\pi (kh)^2 h = \frac{1}{3}\pi r^2 h$.

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(b) The straight line $y = kx$ cuts the curve $(x - k)^2 + y^2 = k^2$ at the point where $x = 0.6$. Find exactly the possible values of y at this point. [3]

(b) $y = kx$ —— (1)

$(x-k)^2 + y^2 = k^2$ —— (2)

Sub (1) and $x = 0.6$ into (2):

$(0.6-k)^2 + (0.6k)^2 = k^2$

$0.36 - 1.2k + k^2 + 0.36k^2 - k^2 = 0$

$0.36k^2 - 1.2k + 0.36 = 0$

$\frac{9}{25}k^2 - \frac{6}{5}k + \frac{9}{25} = 0$

$9k^2 - 30k + 9 = 0$

$3k^2 - 10k + 3 = 0$

$(3k - 1)(k - 3) = 0$

$k = \frac{1}{3}$ or $k = 3$

When $k = \frac{1}{3}$, $x = 0.6$: $y = \frac{1}{3}\left(\frac{3}{5}\right) = \frac{1}{5}$

When $k = 3$, $x = 0.6$: $y = 3\left(\frac{3}{5}\right) = \frac{9}{5}$

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3 (a) One of the roots of the equation $z^3 + pz^2 - z + q = 0$, where p and q are real numbers, is $1 + 2i$. Find the other roots of the equation, the value of p and the value of q. [5]

(a) Since all the coefficients of the cubic equation are real, by complex conjugate root theorem,
$z = 1-2i$ is the other root, and the 3rd root is $z = a$, where $a \in \mathbb{R}$.

$(1+2i)^3 + p(1+2i)^2 - (1+2i) + q = 0$

$-11-2i + p(-3+4i) - 1 - 2i + q = 0$

$(-12 - 3p + q) + (4p - 4)i = 0 + 0i$

comparing coefficients,

$4p - 4 = 0$

$p = 1$

$-12 - 3p + q = 0$

$q = 12 + 3(1)$

$q = 15$

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$z^3 + z^2 - z + 15 = [z - (1+2i)][z - (1-2i)](z - a)$

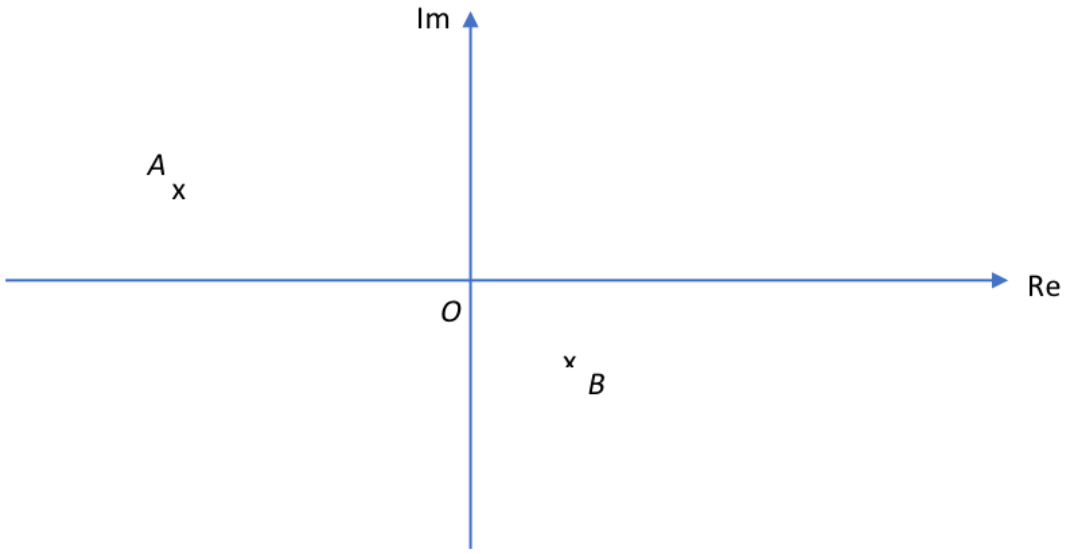
$= (z^2 - 2z + 5)(z - a)$

comparing the constant, $15 = -5a$

$a = -3$

The three roots are $z = 1+2i$, $z = 1-2i$ and $z = -3$

(b) The points A and B on the Argand diagram represent the complex numbers w_1 and w_2 respectively.

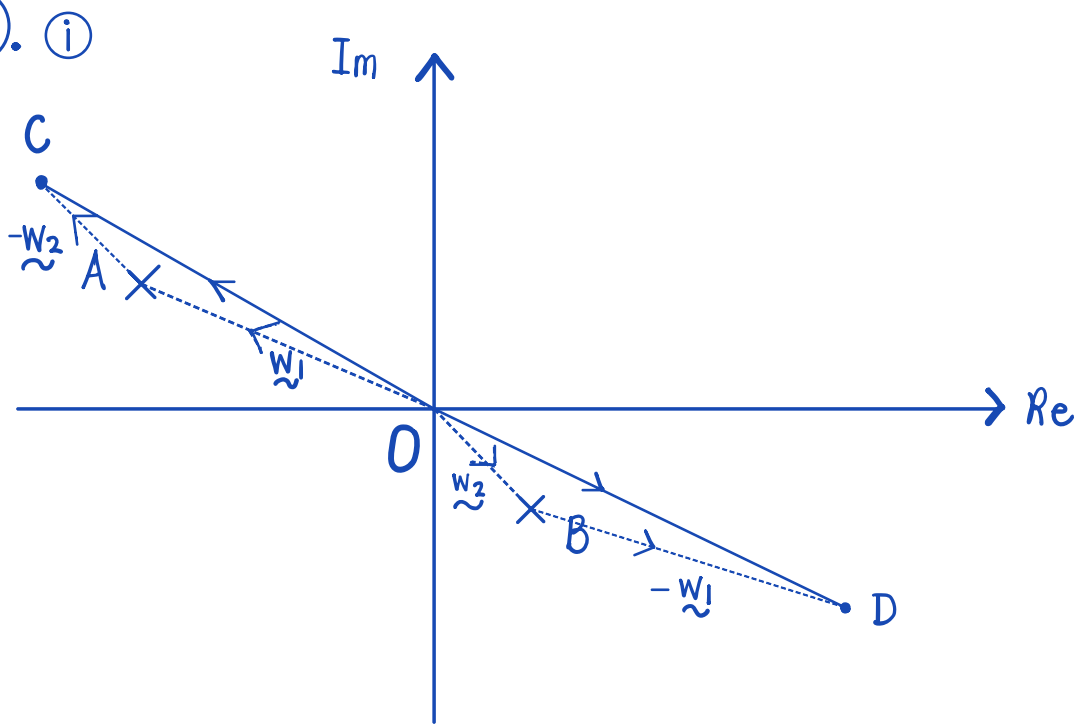


The complex number w_3 is defined as $w_3 = w_1 - w_2$, and the complex number w_4 is defined as $w_4 = w_2 - w_1$.

(i) On the copy of the Argand diagram in the Printed Answer Booklet, mark the point C that represents w_3 and the point D that represents w_4 .

State the geometrical relationship between points C and D. [3]

b. (i)



D is a 180° rotation of C about O.

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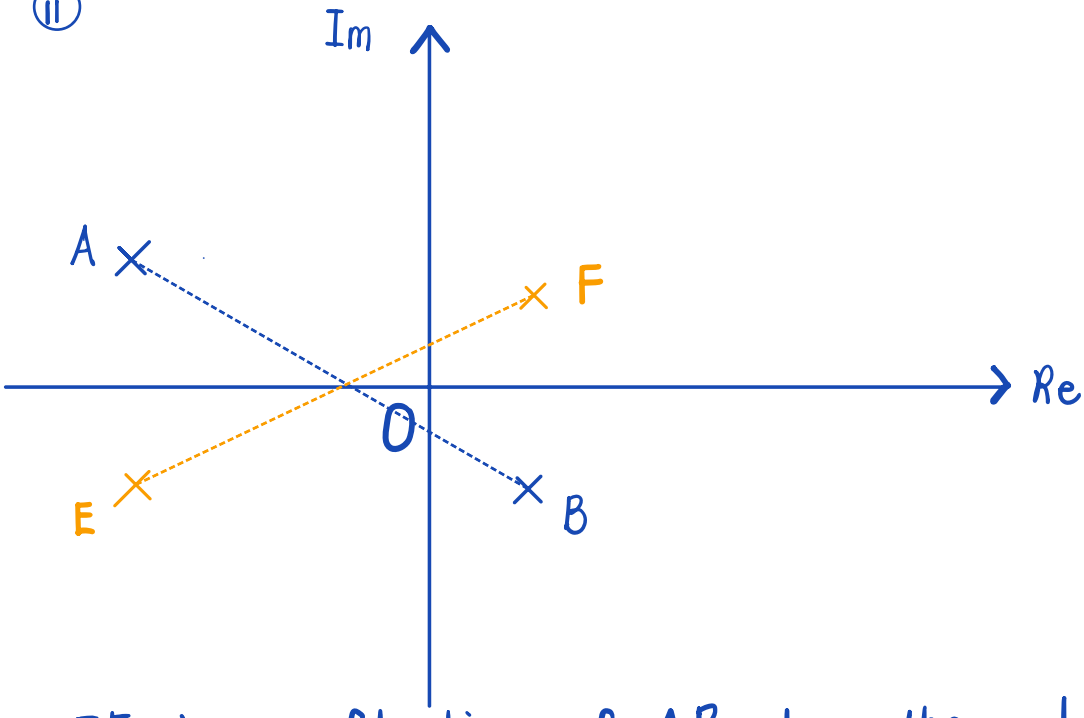
The points E and F represent the complex numbers w_1^* and w_2^* respectively.

(ii) Describe the transformation that maps the line AB onto the line EF. [2]

(ii)



EF is a reflection of AB along the real axis.

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4 An arithmetic series has first term a and common difference d, where $a \neq 0$ and $d \neq 0$.
A geometric series has first term a and common ratio r, where $r > 0$.

The 6th term of the arithmetic series is equal to the 5th term of the geometric series.

The 22nd term of the arithmetic series is equal to the 7th term of the geometric series.

(a) Find the value of r and an expression for d in terms of a.

Describe the set of possible values of a. [5]

(a) Given $a + 5d = ar^4$ —— (1)

$a + 21d = ar^6$ —— (2)

(2) − (1): $16d = ar^6 - ar^4$

$16d = ar^4(r^2 - 1)$

$d = \frac{ar^4(r^2-1)}{16}$ —— (3)

(1): $5d = ar^4 - a$

$d = \frac{ar^4 - a}{5}$ —— (4)

$\frac{ar^4(r^2-1)}{16} = \frac{ar^4 - a}{5}$

$ar^4(r^2-1) = 16a(r^4-1)$

$5r^6 - 5r^4 = 16r^4 - 16$

$5r^6 - 21r^4 + 16 = 0$

By GC: $r = -2$ (rej.), $r = -1$ (rej.), $r = 1$ (rej.) or $r = 2$

(4): $d = \frac{a(2^4) - a}{5}$

$= \frac{15a}{5}$

$\therefore d = 3a$

For every non-zero value of a, the values of $r = 2$ and $d = 3a$ satisfy all the given conditions. Hence, $\{a \in \mathbb{R} : a \neq 0\}$.

(b) The sum of the arithmetic series to 37 terms differs by $12a$ from the sum of the geometric series to n terms. Find the value of n. [4]

(b) $\frac{a(2^n-1)}{2-1} - \left[\frac{37}{2}(2a+36d)\right] = 12a$

$a(2^n-1) - (37a + 666d) = 12a$

$a(2^n-1) - 37a - 666(3a) = 12a$

$a(2^n-1) - 2047a = 0$

$2^n - 1 - 2047 = 0$

$2^n = 2048$

$\therefore n = 11$

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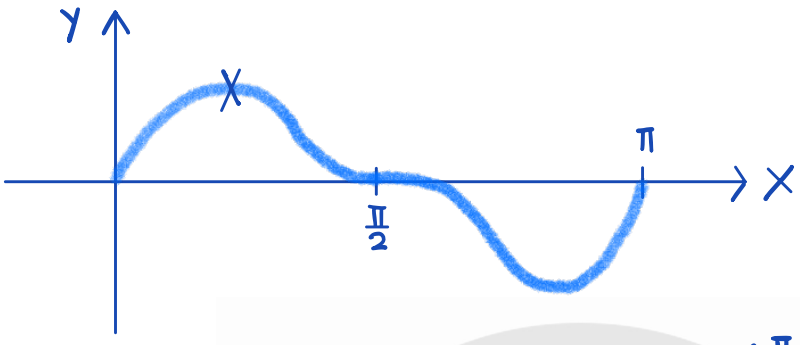
5 (a) Sketch the graph of the curve $y = \cos^2 x \sin 2x$ for $0 \le x \le \pi$, giving the exact coordinates of the points of intersection with the $x-$axis. [2]

(b) Find the exact coordinates of the maximum point. You do not have to show this is a maximum point. [4]

(a)



intersections with the x-axis = $(0,0)$, $(\frac{\pi}{2},0)$ and $(\pi,0)$

(b) $\frac{dy}{dx} = \cos^2 x(2\cos 2x) + \sin 2x \cdot 2\cos x(-\sin x)$

$= 2\cos^2 x \cos 2x - (2\sin x \cos x)(2\sin x \cos x)$

$= 2\cos^2 x(1 - 2\sin^2 x) - 4\sin^2 x \cos^2 x$

$= 2\cos^2 x - 8\sin^2 x \cos^2 x$

$= 2\cos^2 x(1 - 4\sin^2 x)$

Let $\frac{dy}{dx} = 0$

$\therefore 2\cos^2 x = 0$ or $1 - 4\sin^2 x = 0$

$\cos^2 x = 0$ $\quad\quad$ $\sin^2 x = \frac{1}{4}$

$\cos x = 0$ $\quad\quad$ $\sin x = \pm\frac{1}{2}$

$x = \frac{\pi}{2}$ (rej.) $\quad\quad$ $x = \frac{\pi}{6}$

When $x = \frac{\pi}{6}$, $y = (\cos\frac{\pi}{6})^2 \sin 2(\frac{\pi}{6})$

$= (\frac{\sqrt{3}}{2})^2 (\frac{\sqrt{3}}{2})$

$= \frac{3\sqrt{3}}{8}$

Max. pt. = $(\frac{\pi}{6}, \frac{3\sqrt{3}}{8})$

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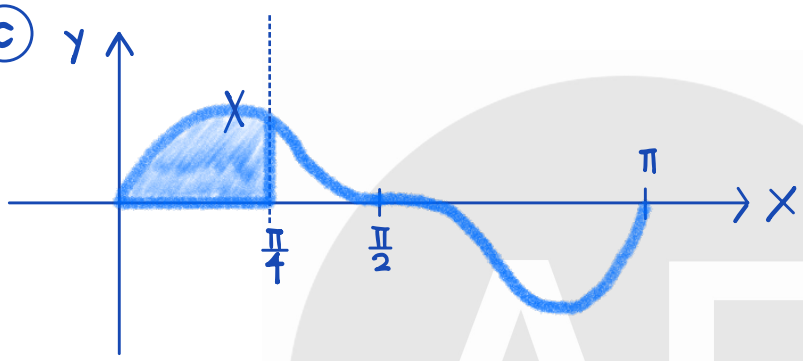
(c) **Do not use a calculator in answering this part.**

Find the exact area bounded by the curve, the $x-$axis and the line $x=0$ and $x=\frac{1}{4}\pi$. [4]



$$\text{Exact area} = \int_0^{\frac{\pi}{4}} \cos^2 x \cdot \sin 2x \, dx$$

$$= \int_0^{\frac{\pi}{4}} \cos^2 x \cdot 2\sin x \cos x \, dx$$

$$= -2\int_0^{\frac{\pi}{4}} \underbrace{(\cos x)^3}_{f(x)^n} \underbrace{(-\sin x)}_{f'(x)} \, dx$$

Recap:

$$\int f(x)^n \cdot f'(x) \, dx = \frac{f(x)^{n+1}}{n+1} + C$$

$$= -2\left[\frac{\cos^4 x}{4}\right]_0^{\frac{\pi}{4}}$$

$$= -2\left[\frac{1}{4}\left(\frac{\sqrt{2}}{2}\right)^4 - \frac{1}{4}(1)^4\right]$$

$$= -2\left[\frac{1}{4}\left(\frac{4}{16}\right) - \frac{1}{4}\right]$$

$$= \frac{3}{8} \text{ sq. units}$$

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Section B: Probability and Statistics [60 marks]

6 The employees of a certain company are put into teams based on the jobs that they do. The company consists of 22 employees from the Red team, 18 from the Blue team, 9 from the Green team and 6 from the Yellow team.

The manager of the company wishes to find out the opinions of the employees about the office facilities provided for employees. Questionnaires are given to all employees but only the 40 employees from the Red team and the Blue team return them.

(a) Explain whether these 40 employees comprise a sample or a population. [1]

The manager decides to form a committee of employees to discuss the results of the questionnaire.

(b) State an advantage of choosing a random sample of employees for the committee. [1]

The manager decides that the committee will consist of 5 employees, such that

- the committee includes an employee from the Green team,
- at least three teams are represented on the committee and
- there are more members from the Red team than from any other single team on the committee.

(c) Find the number of ways in which the committee can be formed. [4]

(a) There are a total of 55 employees from the company, hence these 40 employees are a subset of the population. They form a sample.

(b) In a random sample, every employee has an equal chance of being chosen, so there is no bias-ness from the manager's selection.

(c) Red : 22 Green : 9
Blue : 18 Yellow : 6

case A : R G B R R $= {}^{22}C_3 \times {}^{9}C_1 \times {}^{18}C_1 = 249\,480$

case B : R G Y R R $= {}^{22}C_3 \times {}^{9}C_1 \times {}^{6}C_1 = 83\,160$

case C : R G B Y R $= {}^{22}C_2 \times {}^{9}C_1 \times {}^{18}C_1 \times {}^{6}C_1 = 224\,532$

total number of ways $= 249\,480 + 83\,160 + 224\,532$
$= 557\,172$ ways

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7 (a) A bag contains 20 balls. Some of the balls are red, some are yellow and the rest are blue. Two balls are taken, at random and at the same time, from the bag.
The probability that one of the balls is red and the other one is yellow is $\frac{28}{95}$. The probability that one of the balls is red and the other one is blue is $\frac{4}{19}$.
Determine the number of red balls originally in the bag. [5]

(a) Let r be the number of balls that are red, y be the number of balls that are yellow and b that are blue.

$$\frac{r}{20} \times \frac{y}{19} \times 2! = \frac{28}{95}$$

$$\frac{ry}{190} = \frac{28}{95}$$

$$ry = 56$$

$$y = \frac{56}{r} \quad \text{—①}$$

$$\frac{r}{20} \times \frac{b}{19} \times 2! = \frac{4}{19}$$

$$\frac{rb}{190} = \frac{4}{19}$$

$$rb = 40$$

$$b = \frac{40}{r} \quad \text{—②}$$

$$r + y + b = 20 \quad \text{—③}$$

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Sub ① and ② into ③:

$$r + \frac{56}{r} + \frac{40}{r} = 20$$

$$r^2 + 56 + 40 - 20r = 0$$

$$r^2 - 20r + 96 = 0$$

$$(r - 8)(r - 12) = 0$$

$$r = 8 \quad \text{or} \quad r = 12$$

When $r = 8$, $y = 7$ and $b = 5$

When $r = 12$, $y = \frac{14}{3}$ and $b = \frac{10}{3}$ (rej.)

There are 8 red balls.

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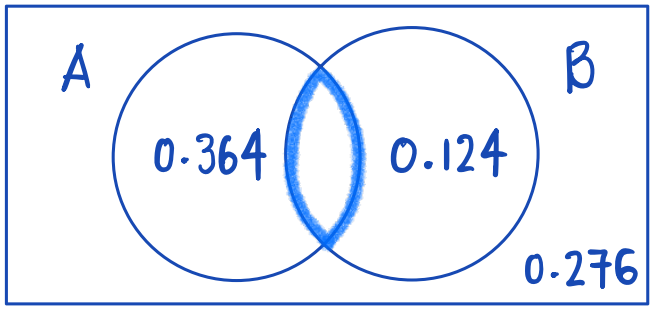
(b) Events A and B are such that $P(A \cap B') = 0.364$, $P(A' \cap B) = 0.124$ and $P((A \cup B)') = 0.276$.

(i) Show that A and B are **not** mutually exclusive events. [2]

(ii) Determine whether A and B are independent events. [2]

(b) i.



$P(A \cap B) = 1 - P(A \cap B') - P(A' \cap B) - P(A \cup B)'$

$= 1 - 0.364 - 0.124 - 0.276$

$= 0.236$

Since $P(A \cap B) \neq 0$, A and B are not mutually exclusive events.

ii. $P(A) = P(A \cap B') + P(A \cap B)$

$= 0.364 + 0.236$

$= 0.6$

$P(B) = P(A' \cap B) + P(A \cap B)$

$= 0.124 + 0.236$

$= 0.36$

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$P(A) \cdot P(B) = 0.6 \times 0.36$

$= 0.216$

Since $P(A \cap B) \neq P(A) \cdot P(B)$,

A and B are not independent events.

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8 Three researchers are investigating the effect of the amount of fertilizer used on crop yields.

(a) Tang uses Fertiliser A. He calculates that the product moment correlation coefficient between the amount of fertilizer used per square metre and the crop yield per square metre is 0.98.

State what else Tang should do before deciding whether a linear model is a good fit to the data. [1]

He should plot a scatter plot between the amount of fertiliser A used per square metre and the crop yield per square metre to observe if they share a linear correlation.

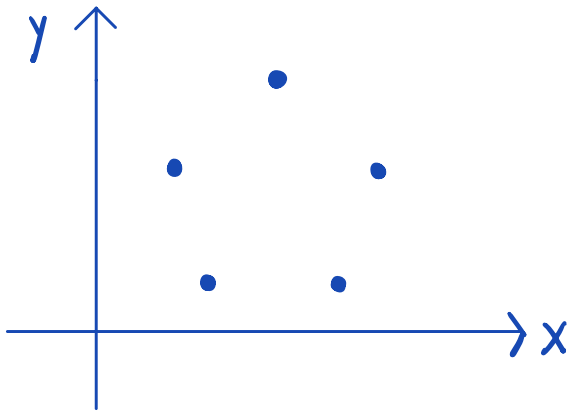
(b) Xi uses Fertiliser B. He finds that the product moment correlation coefficient between the amount of fertilizer used per square metre and the crop yield per square metre is approximately 0.

(i) Explain how Xi should interpret the results of his investigation. [1]

(ii) Sketch a scatter diagram with at least 5 points to illustrate a product moment correlation coefficient of approximately 0. [1]

i. It would mean that the amount of fertiliser B used per square metre and the crop yield per square metre have no linear correlation. But there is a possibility they could still have other forms of relationship.

ii.



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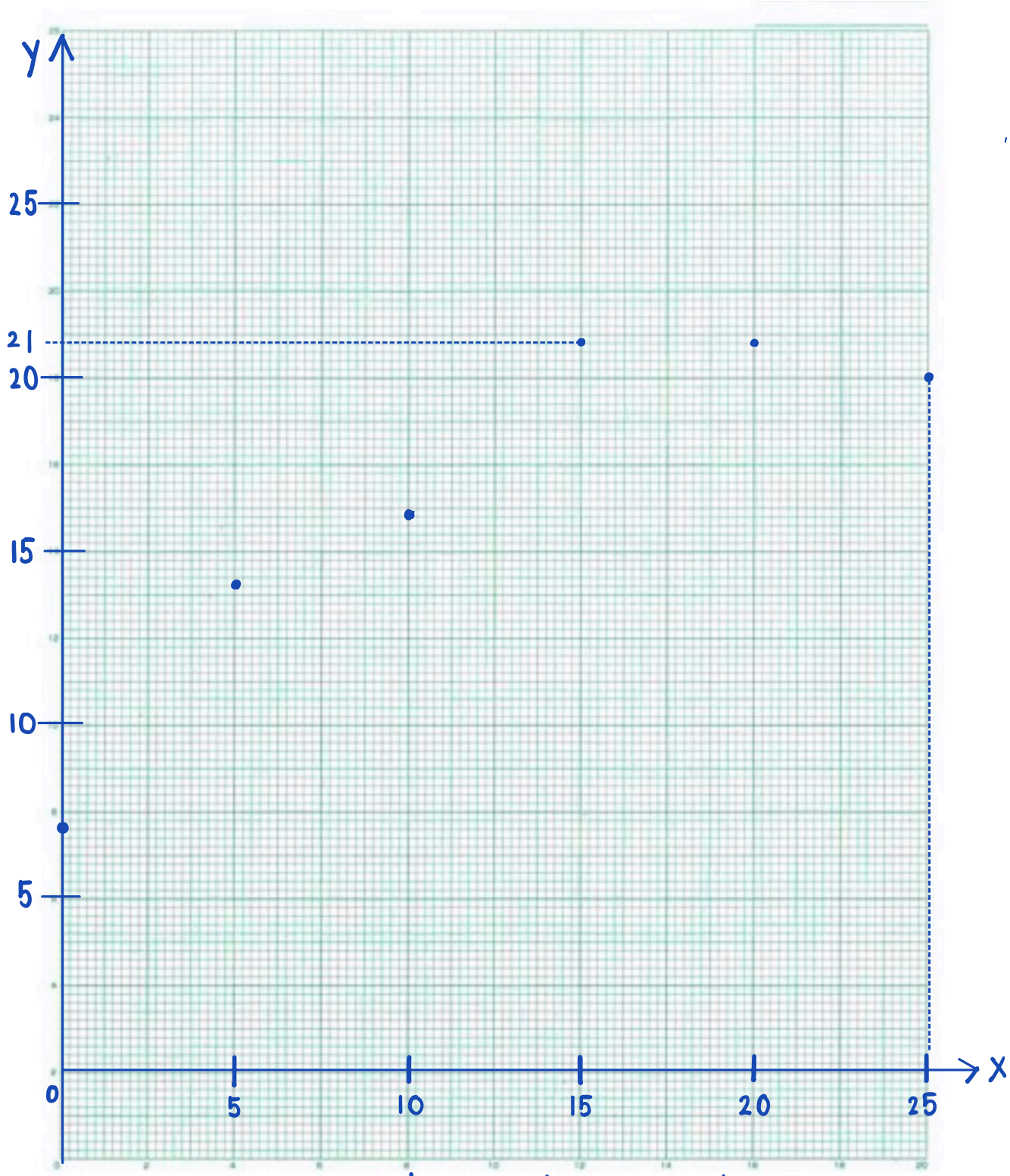
(c) Leia uses Fertiliser C. She sets up 6 test plots of land. The table shows the amount of fertilizer, x grams, used per square metre of each plot and the crop yield, y kilograms, per square metre of each plot.

x	0	5	10	15	20	25
y	7	14	16	21	21	20

(i) Sketch the scatter diagram for Leia's data, and explain how you know that the crop yield is not linearly related to the amount of fertilizer used. [2]

i.



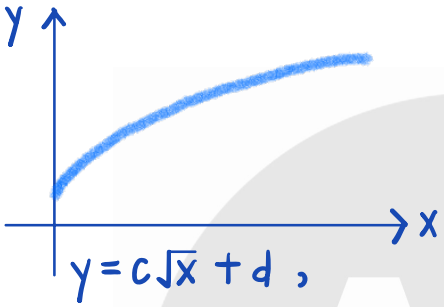
From the scatter plot, we observe that y is increasing at a decreasing rate as x increases.

(ii) By drawing sketches of the equations below, where c, d, e and f are positive constants, explain which is a better model for the relationship between the amount of fertilizer used and the crop yield.

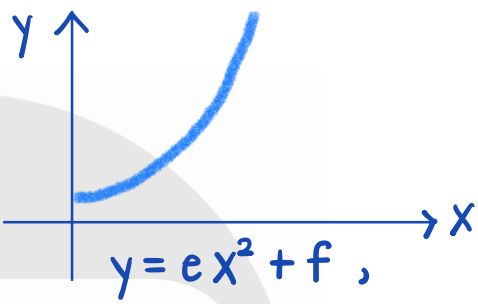
$y = c\sqrt{x} + d$ $\qquad$ $y = ex^2 + f$ [3]

(iii) Use the better model to find an estimate for the crop yield per square metre when the amount of fertilizer used is 13 grams per square metre. Give your answer correct to 3 significance figures. [3]

ii.



$y = c\sqrt{x} + d$,
$c > 0$, $d > 0$



$y = ex^2 + f$,
$e > 0$, $f > 0$

$y = c\sqrt{x} + d$ is the better model because y is also increasing at a decreasing rate as x increases.

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iii.

x	0	5	10	15	20	25
y	7	14	16	21	21	20
$\sqrt{x}$	0	2.24	3.16	3.87	4.47	5

By GC : $y = 2.9113\sqrt{x} + 7.4055$

when $x = 13$, $y = 2.9113\sqrt{13} + 7.4055$

$= \underline{17.9 \text{ kg}}$ per square metre.

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9 Tringos are sweets that are made in four different colours: red, yellow, purple and green. They are sold in packets containing 12 randomly chosen sweets.

(a) A student wishes to estimate the proportion of Tringos that are green. She checks the contents of 100 randomly chosen packets of Tringos and finds the information summarised in the table.

number of green sweets in a packet	0	1	2	3	4	5	6 or more
frequency	15	32	27	18	6	2	0

Use the information in the table to estimate the probability that a randomly chosen Tringo is green. [2]

The probability that a randomly chosen Tringo is red is 0.3.

(b) Find the probability that a packet of 12 Tringos contains fewer than 5 red sweets. [1]

(c) Find the probability that at least 4 out of 7 randomly chosen packets of Tringos contain fewer than 5 red sweets. [2]

(a) $\dfrac{(0\times15)+(1\times32)+(2\times27)+(3\times18)+(4\times6)+(5\times2)}{(100\times12)}$

$= \underline{0.145}$

(b) Let X be the r.v. denoting the number of red Tringos in a packet of 12.

$X \sim B(12, 0.3)$

$P(X<5) = P(X\leq4) = 0.72366$

$= \underline{0.724}$

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(c) Let Y be the r.v. denoting the number of packets of Tringos out of 7, that contain fewer than 5 red sweets.

$Y \sim B(7, 0.72366)$

$P(Y\geq4) = 1 - P(Y\leq3)$

$= 0.90255$

$= \underline{0.903}$

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Tringos are delivered to shops in cartons containing 72 randomly chosen packets of 12 sweets.

(d) Find the probability that there are more than 250 red sweets in a randomly chosen carton of Tringos. [2]

(e) Given that the probability that a randomly chosen packet of 12 Tringos contains either 2 or 3 yellow sweets is 0.206, find the **two** possible values of p. Give your values correct to 3 decimal places. [3]

(d) total number of sweets $= 72 \times 12$
$= 864$

Let W be the r.v. denoting number of red Tringos out of 864.

$W \sim B(864, 0.3)$

$P(W > 250) = P(W \geq 251) = 1 - P(W \leq 250)$
$= 0.73987$
$= \underline{0.740}$

(e) Let V be the r.v. denoting number of yellow Tringos in a packet of 12.

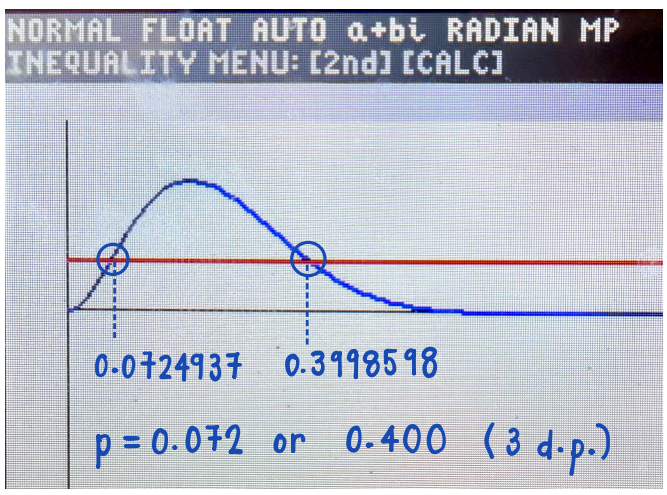
$V \sim B(12, p)$

$P(V=2) + P(V=3) = 0.206$

$\binom{12}{2} p^2 (1-p)^{10} + \binom{12}{3} p^3 (1-p)^9 = 0.206$

$66p^2(1-p)^{10} + 220p^3(1-p)^9 = 0.206$

By GC:



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10 In this question you should assume that the distributions are all independent of each other.

Mrs Ng lives in Aytown. She often drives to the neighbouring towns of Beetown and Seatown.

(a) The time taken, in minutes, for Mrs Ng to drive from Aytown to Beetown using her usual route has the distribution $N(22, 3^2)$.

(i) Find the probability that Mrs Ng takes between 22.5 and 24 minutes for a randomly chosen journey from Aytown to Beetown. [1]

Mrs Ng often visits the shops in Beetown. The time, in minutes, she spends at the shops has the distribution $N(75, 12^2)$. The time taken for the journey from Beetown to Aytown also has the distribution $N(22, 3^2)$.

(ii) Find the probability that a randomly chosen trip from Aytown to the shops at Beetown and back to Aytown by Mrs Ng takes less than 2 hours. [3]

The time taken, in minutes, for Mrs Ng to drive from Aytown to Beetown using an alternative route has the distribution $N(25, 4^2)$.

(iii) Find the probability that a randomly chosen journey from Aytown to Beetown using the alternative route takes less time than a randomly chosen journey using Mrs Ng's usual route. [3]

(a) i. Let X be the r.v. denoting the time taken, in minutes, for Mrs Ng to drive from Aytown to Beetown.

$$X \sim N(22, 3^2)$$

$$P(22.5 < X < 24) = 0.18132$$
$$= \underline{0.181}$$

ii. Let Y be the r.v. denoting the time, in minutes, she spends at the shops in Beetown.

$$Y \sim N(75, 12^2)$$
$$X_1 + Y + X_2 \sim N(119, 162)$$

$$P(X_1 + Y + X_2 < 120) = 0.53131$$
$$= \underline{0.531}$$

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iii. Let W be the r.v. denoting the time taken, in minutes, for Mrs Ng to drive from Aytown to Beetown using the alternative route.

$$W \sim N(25, 4^2)$$
$$X - W \sim N(-3, 5^2)$$

$P(X > W)$

$= P(X - W > 0)$

$= 0.27425$

$= \underline{0.274}$

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(b) The time taken, in minutes, for the journey from Aytown to Seatown is also normally distributed. The probability that a randomly chosen journey takes less than 55 minutes is 0.873 and the probability that a randomly chosen journey takes more than 49 minutes is 0.471.

Find the probability that a randomly chosen journey from Aytown to Seatown takes between 45 and 50 minutes. [4]

Let V be the r.v. denoting the time taken, in minutes, for the journey between Aytown to Seatown.

$$V \sim N(\mu, \sigma^2)$$

$P(V < 55) = 0.873$

$P\left(Z < \frac{55-\mu}{\sigma}\right) = 0.873$

$\frac{55-\mu}{\sigma} = 1.140687475$

$\mu = 55 - 1.140687475\sigma$ —(1)

$P(V > 49) = 0.471$

$P\left(Z > \frac{49-\mu}{\sigma}\right) = 0.471$

$\frac{49-\mu}{\sigma} = 0.072756351\,1$

$\mu = 49 - 0.072756351\,1\sigma$ —(2)

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$\therefore 55 - 1.140687475\sigma = 49 - 0.072756351\,1\sigma$

$1.067931124\sigma = 6$

$\sigma = \underline{5.6183}$

$\therefore \mu = 55 - 1.140687475(5.6183)$

$= \underline{48.591}$

Hence, $P(45 < V < 50) = 0.33765$

$= \underline{0.338}$

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11 (a) A company uses a machine to pack bird seed into packets. Packets should contain a mean mass of 400 grams of seed. Following maintenance to the packing machine, the owner of the company wishes to check if packets contain a mean mass of 400 grams of seed. He decides to carry out a hypothesis test at the 5% level of significance.

The owner finds the masses, x grams, of bird seed in a random sample of packets. Summary data is as follows.

$n = 40$ $\quad \Sigma(x - 400) = 120$ $\quad \Sigma(x - 400)^2 = 1902$

(i) Write down the hypotheses for the test, defining the parameter(s) you use. [2]

(ii) Find the critical region for the test, giving your answer in grams correct to 1 decimal place. [4]

(iii) State the conclusion of the test in the context of the question. [2]

(a) i. To test $H_0 : \mu = 400g$
against $H_1 : \mu \neq 400g$ at 5% level of significance.

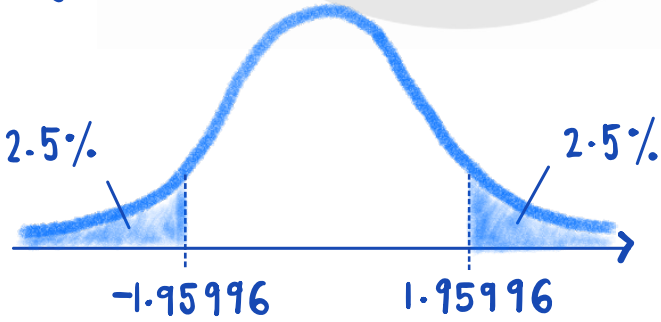
where H_0 = null hypothesis,
H_1 = alternate hypothesis,
μ = population mean.

ii. $\bar{x} = \frac{\Sigma(x-400)}{n} + 400$
$= \frac{120}{40} + 400 = 403g$

$s^2 = \frac{1}{n-1}\left[\Sigma(x-400)^2 - \frac{(\Sigma(x-400))^2}{n}\right]$
$= \frac{1}{39}\left[1902 - \frac{120^2}{40}\right]$
$= 39.538$

Let X be the r.v. denoting the mean mass of a packet of seeds.

since n is large enough, by CLT, $\bar{X} \sim N(400, \frac{39.538}{40})$ approximately



For a two-tailed test, at 5% level of significance,
H_0 is rejected when z-value ≤ -1.95996 or z-value ≥ 1.95996

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i.e $\dfrac{\bar{x} - 400}{\sqrt{\frac{39.538}{40}}} \leq -1.95996$ or $\dfrac{\bar{x} - 400}{\sqrt{\frac{39.538}{40}}} \geq 1.95996$

$\bar{x} - 400 \leq -1.9486$ or $\bar{x} - 400 \geq 1.9486$

$\bar{x} \leq 398.051$ or $\bar{x} \geq 401.9486$

$\bar{x} \leq 398.1$ or $\bar{x} \geq 401.9$

iii. Since $\bar{x} = 403$ g, it lies within the critical region.
We reject H_0, i.e. the mean mass of a packet of bird seed is not 400g.

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(b) The company also has a machine that fills bags with pet food. The mean mass of pet food is stated to be 500 grams per bag. Following comments from customers, the owner of the company decides to carry out a z-test using

$H_0: \mu = 500$

$H_1: \mu > 500.$

(i) State what the owner is testing for. [2]

(ii) State the critical z-value used for this test when it is carried out at the 5% level of significance. [1]

The machine is used each week from Monday to Friday. The owner decides to take the last 5 bags filled on Monday, Wednesday and Friday one week, to make up a sample of 15 bags for his test.

(iii) Give **two** reasons why the owner's sample is **not** suitable for this test. [2]

(i) The owner is testing for the claim that the mean mass of pet food could be more than what is stated on the bag.

(ii) critical $z-$value $= 1.6449$
$= \underline{1.64 \text{ (3 s.f.)}}$

(iii) 1. The sample is <u>not representative</u> of the bags filled by the machine from Monday to Friday, as no bags from Tuesday or Thursday were collected to form the sample.

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2. The <u>sample size is small</u>. Central Limit Theorem cannot be applied to approximate the distribution of the sample mean to be normally distributed.

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